OPTIMUM HULL SPACING OF A FAMILY OF MULTIHULLS

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Abstract

Multihull configurations are optimised for minimum wave-making using thin-ship theory. Only the number of hulls, and their placement and beams are varied, the total displacement of the vessel and the individual hull length and shape being fixed. In particular, optimum configurations are determined for two, three and four-hulled vessels, with and without longitudinal stagger. An example of use of this theory is given, for a family of multihulls relevant to a high-speed ferry design. Competing designs within this family are assessed over a large speed range, with respect to generated wave amplitude, wave resistance, and total drag.

Keywords: multihulls, catamarans, ship waves, wave resistance.

1 Introduction

The aim of this paper is to optimise the *placement* of individual hulls in a multihull configuration, in order to minimise wavemaking. No attempt is made here to optimise the *shape* of individual hulls. For a general multihull vessel, these are coupled tasks, but we consider here only a special case where they are uncoupled.

The main content of the present paper is a detailed theoretical treatment of this wave minimisation problem. However, in order to motivate this theory from the practical point of view, we first provide samples of results that follow from it. These results are for a vessel whose dimensions are of reasonable relevance to a specific design purpose, namely a small high-speed passenger ferry. We fix both the total displacement and a measure of the useable deck area. Then we assemble an array of hulls in various configurations suggested by the theoretical analysis, and contrast the performance of the whole vessel on three criteria, namely maximum generated wave height (within a certain prescribed patch aft of the vessel), wave resistance, and total drag. This is done over a large range of speeds, allowing decisions to be made on the best multihull configuration at each speed, on any of the three criteria.

The multihull vessels considered here have individual hulls which are identical except for a simple scaling of offsets. That is, they have the same length, draft, and shape, but can have different beams. In that case, according to thin-ship theory, the wavemaking properties of the individual hulls are identical aside from a constant multiplier. Linear superposition of the far-field free-wave patterns then leads to a combined wave spectrum which is the product of two distinct factors, one representing the effect of hull shape and the other the effect of hull placement, and we can (almost) ignore the former factor, and concentrate our attention on minimising the latter factor. This factor represents wave interference effects, where one hull cancels waves made by another.

Each factor in the combined wave spectrum is a function of the angle θ of wave propagation, measured to port of the direction of travel of the vessel. When we minimise wavemaking, we do so for some range of values of θ. Hence there is still a small amount of indirect coupling between hull shape and hull placement, in determining that range. For example, there is little point in choosing a hull placement that cancels waves near to some particular angle $\theta = \theta_0$ if we are going to use individual hulls that make no waves at that angle.

However, anticipating the need to accommodate a variety of hull shapes, we set ourselves here the task of minimising waves by maximising hull-hull cancellation over the widest possible range of θ values. In particular, we always seek if possible to cancel out pure transverse waves with $\theta = 0$ whose crests are exactly perpendicular to the ship's track. This is hardly a new concept in ship design! From the days of Froude or earlier, naval architects have always tried to operate in regimes where such transverse-wave cancellation (notionally between bow and stern for monohulls) is maximised. This leads to hollows (local minima) in the wave resistance curve, for example. In the same spirit, here we *exactly* cancel the pure transverse waves, wherever it is possible to do so by careful longitudinal placement of hulls.

Cancellation of non-transverse waves is not such an easy task, however, and depends essentially on a careful choice of the lateral separation distances. Again, exact cancellation is possible at any fixed wave angle $\theta = \theta_0 > 0$, and we give examples of side-by-side multihulls where this is achieved at useful angles θ_0. The required lateral separations become large when θ_0 is small, and this would happen if we were to attempt vainly to cancel transverse waves by lateral separation only. Conversely, the required lateral separations become small when θ_0 approaches $90°$. In practice, θ_0 (as a measure of the wave angle where cancellation is most desirable) increases with speed. Hence above a certain speed, the optimum lateral separations have become so small that little further can be done by hull placement to cancel waves, and success in wave reduction (for patterns dominated by extreme diverging waves, with crests nearly parallel to the ship's track) must come from hull shape design.

There is a sense in which multihulls are always superior to monohulls from the point of view of wave reduction, and in particular wave resistance reduction. After all, since wave resistance varies as beam squared, the total wave resistance of two separate half-beam hulls is half of that of one full-beam hull! Hence the large-separation limit for identical side-by-side hulls will always produce formally a lower wave resistance (by a factor equal to the reciprocal of the number of hulls) than the small-separation limit. What we seek is an even better result for some finite separation. However, this is not always achievable, and in practice the optimum hull spacings are sometimes infinite. In that case, the actual spacings can be chosen on grounds other than minimisation of wavemaking, providing that the separation between hulls is sufficient to achieve a wave resistance not excessively greater than that for infinite separation.

2 An example design

Suppose our task is to design a vessel of total displacement 31.25 tonnes, using individual hulls of a Wigley shape [9] that are each of length 19.1 metres and draft 1.25 metres. We then have freedom only to choose the number of hulls, their individual beams, and their locations.

2.1 Some preliminaries

Before examining some multihull arrangements in detail, it is instructive to consider the wave-generating characteristics of Wigley monohulls, which have parabolic sections and waterlines, and a rectangular side view.

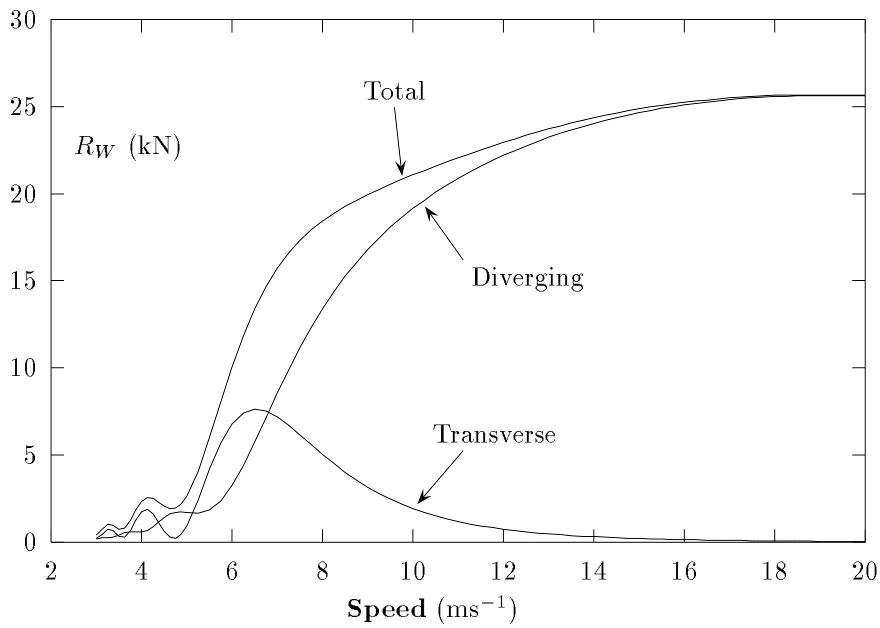

Figure 1: Wave resistance of a Wigley monohull.

Figure 1 shows the wave resistance of a 31.25t Wigley monohull of length 19.1m and draft 1.25m over a wide range of speeds. We have no need for non-dimensionalisation, since our computations are for a specific vessel size, and therefore show the actual wave resistance force in kiloNewtons (kN), as a function of speed in metres per second (ms^{-1}). Figure 1 gives the total wave resistance as well as that portion which is separately due to transverse and diverging waves. Here, transverse waves are defined ([7] p.273) to be those propagating between $\theta = 0$ and $|\theta| = \arcsin(1/\sqrt{3}) = 35.3°$; diverging waves propagate at angles greater than 35.3°.

Clearly, if we are designing a vessel for a specific speed, it is important to employ methods of wave cancellation that are appropriate to that design speed. Thus for a high-speed vessel, operating at speeds typically greater than about 7 ms^{-1} for this example, it is of paramount importance to eliminate diverging waves. On the other hand, a vessel designed to spend most of its time at relatively low speeds, typically less than 7 ms^{-1}, should employ means to reduce both transverse and diverging waves.

As mentioned in the Introduction, cancellation of diverging waves is a difficult task and depends on a careful choice of lateral hull separation. At some speeds, minimum wavemaking can occur for hulls that are in theory infinitely far apart; at some other speeds, the optimal configuration may be one where the individual hulls are impractically close together. Thus it is necessary to fix at least one more parameter in order to eliminate unreasonably wide or narrow configurations.

We choose to fix at 171.9m^2 a notional measure of the "deck area" of the vessel, namely the area of the polygon that encloses the endpoints of all hulls. So, for example, a conventional 19.1m long catamaran will have the centreplanes of its hulls 171.9/19.1 = 9.0 metres apart; it is consistent with this interpretation of deck area that we say that this vessel has "width" 9.0m, ignoring the contribution of the beam of the individual hulls. We reject the possibility of use of a monohull vessel for the present purpose, where this definition of deck area would be inappropriate; in any case, a 19.1m long monohull is not a competitive 31.25t vessel at the speeds of interest here.

It should be emphasised that the present example is not intended as a practical design. In particular, the Wigley hull is chosen purely for mathematical convenience and because it is a well-known benchmark hull for wave resistance purposes, but conveys no real-world benefits; indeed, it may be particularly bad for some purposes. Our only aim here is to contrast various possible multihull arrangements, in a size and speed regime that is not too different from what may be of interest for real vessel design. However, with a more practical hull shape, and inclusion of further design constraints, the principles outlined here could form the basis for a real design.

2.2 Specifications of vessels

The vessels considered are as follows, sketches and indications of the notional deck areas being provided in accompanying Figure 2. The beams are relatively exaggerated in this figure for greater clarity.

CAT: A conventional side-by-side catamaran of overall length 19.1m, the two hulls each having beam 1.47m with centreplanes 9.0m apart.

WEI: An unconventional laterally-unsymmetrical two-hulled vessel intended to provide transverse-wave cancellation at about 10ms^{-1} via longitudinal

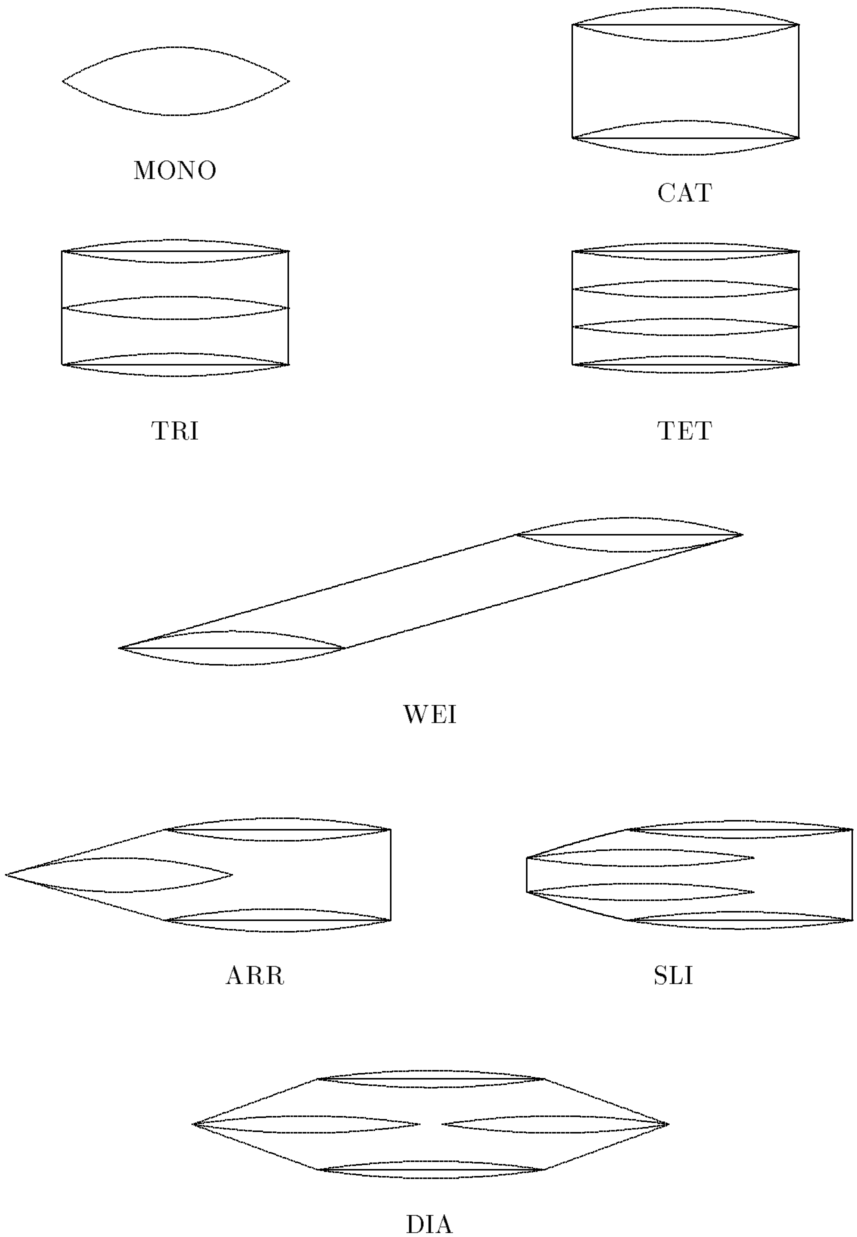

Figure 2: Vessel planforms.

stagger, as well as diverging-wave cancellation to starboard at that speed. This vessel (called a “Weinblum” by Söding [10]) has to have a rather extreme overall length of 51.2m in order to provide the requisite stagger and deck area.

TRI: A vessel consisting of three identical hulls each of beam 0.98m, side by side 4.5m apart, so that the overall length and width is the same as the CAT.

ARR: A three-hull vessel in an arrow formation, the central hull being larger, with half the displacement. The half-angle of the arrow is 15.1°, see later. There is longitudinal stagger such that transverse waves are cancelled at 6.3ms^{-1}, and the resulting overall length of this vessel is 31.6m.

TET: A vessel consisting of four identical hulls each of beam 0.74m, side by side 3.0m apart; again the overall length and width is as for the CAT.

SLI: A four-hull vessel in a blunt arrow configuration, all four hulls being identical, the forward two hulls being 2.82m apart and the aft two being 8.66m apart. The longitudinal stagger is such as to cancel transverse waves at 5.8ms^{-1}, resulting in a length overall of 29.9m. This type of vessel is sometimes called a “Slice” [8].

DIA: A four-hull vessel in a diamond configuration, all four hulls being identical, the outer hulls being 5.88m apart. The two central hulls are nearly nose-to-tail, the overall length being 39.3m. This vessel yields almost total wave cancellation at about 5.5ms^{-1}.

2.3 Wave amplitudes

We use Michell's thin-ship theory (see later) to construct the free-wave spectra for each vessel, and then compute the actual free-wave patterns over an area aft of the vessel where the amplitudes may be expected to be largest and of most environmental concern. Namely, we survey a sectorial patch extending from 47.7m (i.e. 2.5 times the hull length) to 225.6m from the stern of the aftmost hull, and in angle from the centreline track to just beyond the Kelvin 19.5° angle. The values in metres quoted for the maximum wave amplitude are averages of the largest elevation and the deepest trough size observed in such a patch, which usually occur near the corner which is nearest to the ship and to the Kelvin angle.

Our wave-pattern computations are based on an algorithm given in Appendix C of [11], which carries out the numerical integration in a manner consistent with the method of stationary phase, but is not limited to the extreme far field. It is only necessary that we be far enough aft of the ship that the local part of the flow field around the hull is negligible, and it is usual to assume that this is a satisfactory approximation beyond about one or two shiplengths.

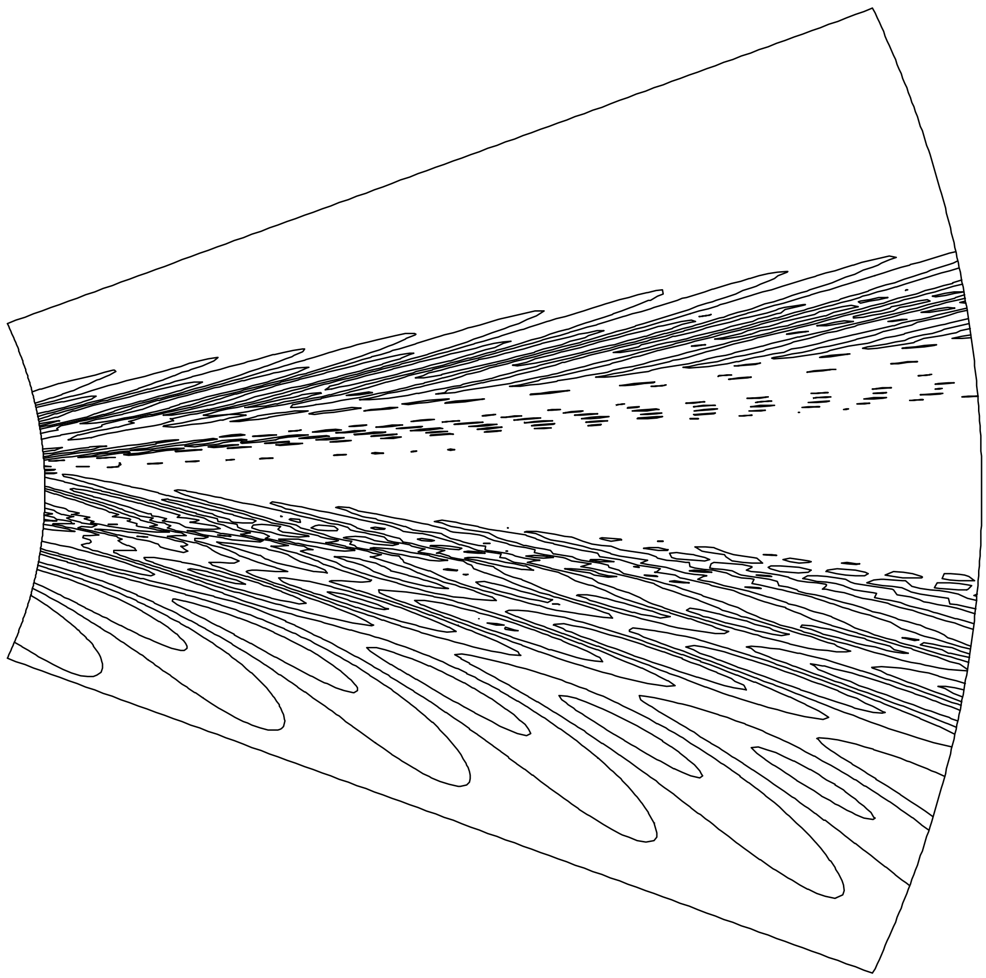

Figure 3: WEI wave contourplot for $U = 10.0\mathrm{ms}^{-1}$.

An example wave contour computed by this method is given in Figure 3. This Figure shows the asymmetry of the wave pattern produced by the WEI vessel (c.f. [10]). The longitudinal stagger for this vessel is such that transverse waves are cancelled totally at $10\mathrm{ms}^{-1}$, and Figure 3 confirms this feature, by virtual absence of disturbance along the ship's track. To starboard of the ship's track, there are narrow troughs and crests lying along almost straight lines, which indicates that the waves are propagating in one dominant direction. The crest lines are quite acute to the ship's track, corresponding to large wave propagation angles $|\theta|$, of the order of $65° - 75°$. Therefore, on the starboard side, there is almost total cancellation of waves propagating at angles less than 65°, which is in agreement with the theoretical analysis presented later. On the port side of the vessel, the contours show gently-curving broad crests and troughs combining to give a conventional paisley-like ship-wave pattern, in which curvature in the crests and troughs indicates that the waves are propagating over a wide range of angles. Interestingly, for this speed, the actual maximum and minimum wave amplitudes are almost the same on either side of the vessel, because even though there has been dramatic cancellation of low-θ waves on the starboard side, there is still significant energy in the highly-divergent waves.

Figure 4 shows the maximum wave amplitude (in metres) within the surveyed patch, for each of the candidate hulls, as a function of speed in metres per second. There are two curves for the laterally unsymmetric WEI vessel, since this vessel produces different waves on its starboard side than its port side. There is a lack of smoothness about some of these curves, due mainly to difficulties in estimation of the maximum wave elevations from computed data in the specified patch. The computational effort to produce these results is quite significant, every point on the curves of Figure 4 demanding data on a grid as dense as that used to produce Figure 3. In particular, whenever (as on the starboard side of Figure 3) there are narrow crests and troughs corresponding to extreme diverging waves, this presents difficulties in estimation of maximum elevations. Nevertheless, we have confidence that the "true" wave amplitudes will be simply smoothed versions of the curves presented here.

These multihull vessels all produce relatively small waves, of amplitude less than 0.55m in the present patch area at any speed (slightly higher for the CAT at very high speed), and less than half of what an equivalent monohull would produce at most speeds. The TET, SLI and TRI produce waves of amplitude less than 0.35m for all speeds up to $14\mathrm{ms}^{-1}$ or higher. The TET performs particularly well at moderately high speeds, making waves of amplitude less than 0.25m at speeds between 9 and $14\mathrm{ms}^{-1}$. However, the TET and TRI still make 0.3m waves even when the speed is as low as about $6\mathrm{ms}^{-1}$, whereas the staggered vessels make much smaller waves at such low speeds, typically 0.05 to 0.15m.

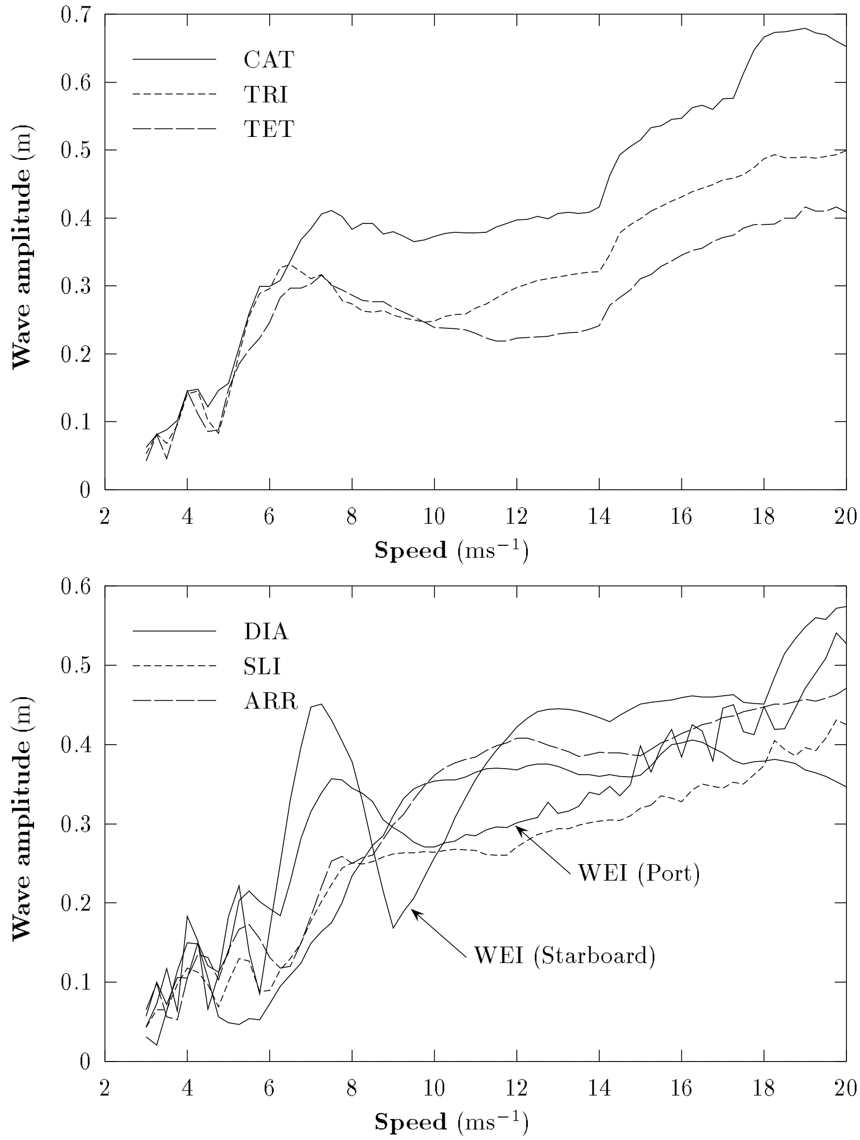

Figure 4: Wave amplitude versus speed.

The two curves for the WEI each have a maximum at a speed of about 7ms^{-1}, the starboard wave amplitude then being a relatively large 0.45m. On the other hand, at 9ms^{-1}, the wave amplitude on the starboard side reaches a minimum of only about 0.15m, by far the best of all the candidate vessels at that speed. However, this extreme wave cancellation occurs for only a small range of speeds, and even small excursions from the optimum speed lead to a rapid increase in wave amplitude on the starboard side of the ship. On the port side, the wave amplitude is higher than on the starboard side for speeds between 8ms^{-1} and 10ms^{-1}. For all other speeds greater than about 6ms^{-1}, the waves on the port side are smaller than on the starboard side, although there is never an extreme degree of port-side cancellation.

Similarly, the DIA vessel makes by far the smallest waves (almost none in fact, with an amplitude as low as 0.04m) at its design speed of about 5.5ms^{-1}, in agreement with the theory to be presented later. This performance is less sensitive to speed than for the WEI's starboard waves, and of course is now achieved on both sides of this symmetric vessel.

2.4 Wave resistance

The same Michell [5] procedure used to calculate wave heights yields the actual wave resistance, simply by integration of the free-wave spectrum with respect to the angle of propagation, see later.

Figure 5 shows the wave resistance of the candidate vessels. For relatively low speeds, from about 4.75ms^{-1} to 9.0ms^{-1}, where Figure 1 shows that transverse wave production by individual hulls is significant, vessels with longitudinal stagger (ARR, SLI and DIA) have the lowest wave resistance, because they use hull-hull interference to cancel these transverse waves. The best of all in this speed range is clearly the DIA, which displays well in Figure 5(b) its design (see later) for extremely low wavemaking at 5.5ms^{-1}, but also performs reasonably well at other speeds and in particular seems again to become the best vessel at the highest speed considered here, 20ms^{-1}.

The side-by-side variants, (CAT, TRI and TET) exhibit relatively large wave resistance in the low-speed range, and TRI and TET possess peaks of magnitude about 9kN at 6.75ms^{-1}. Evidently these vessels are doing little to reduce the transverse waves, as is clear from the fact that the transverse wave drag shown in Figure 1 for the monohull at 6.75ms^{-1} is approximately 8kN.

For speeds greater than about 10ms^{-1}, where transverse wave drag is

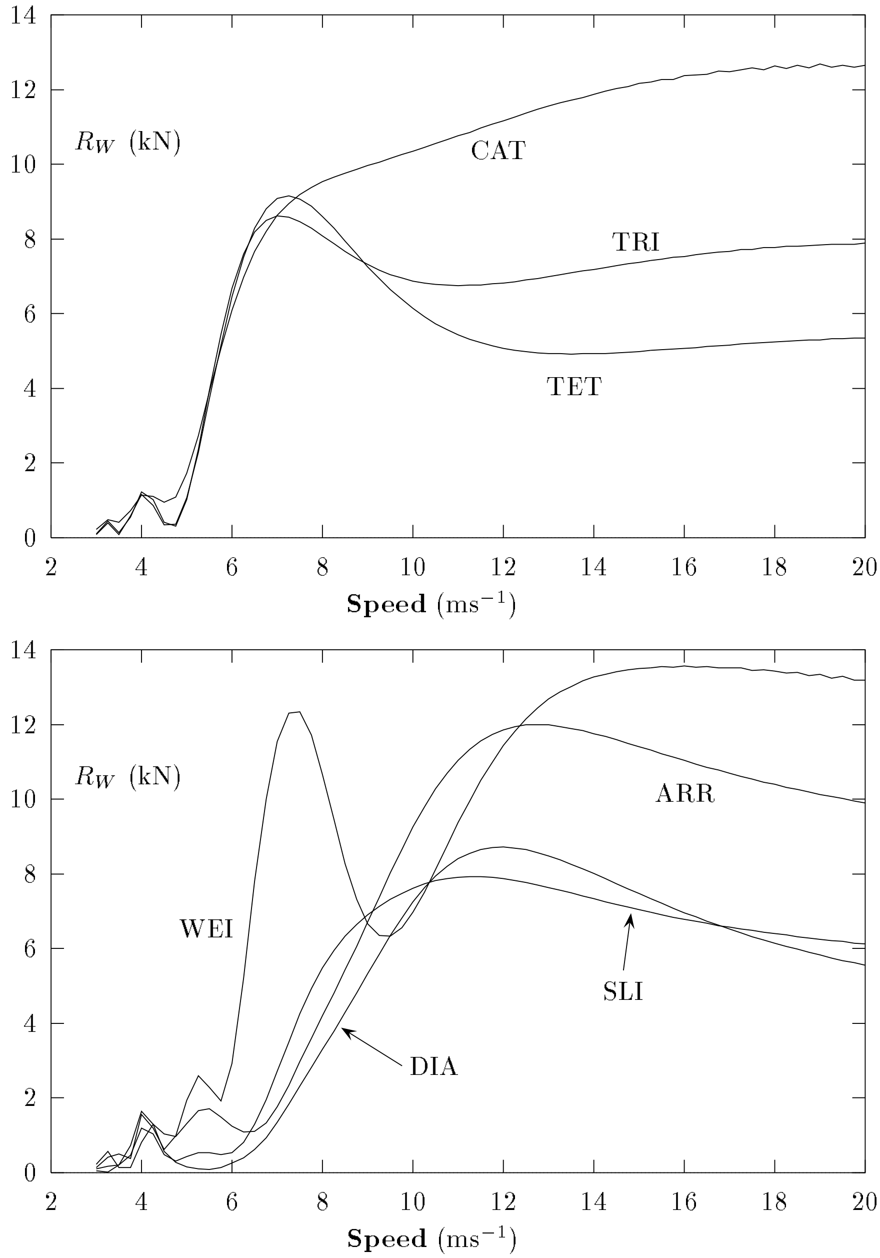

Figure 5: Wave resistance versus speed.

less important, the three four-hulled vessels have the lowest wave resistance, the two trihulls are next best, and the two dihulls have the highest wave resistance. The TET vessel is remarkably good at high speeds, having clearly the lowest wave resistance (less than 6kN) for speeds between 10 and 20ms^{-1}.

The Weinblum dihull vessel WEI happens to be the best of all at a speed of approximately 9.5ms^{-1}. However, slight deviations from this optimum speed lead to large increases in wave drag, and the speed range in which WEI is the best vessel of this set is very narrow. There are two main peaks in the wave resistance curve for this design; one of approximately 12kN at 7.3ms^{-1} and one of approximately 13kN at 16ms^{-1}. For the other staggered variants (ARR, SLI and DIA), the peak wave resistance (of less than 8kN) occurs at speeds of around 11 to 13ms^{-1}.

Generally there is a good consistency between the qualitative features of the wave-resistance curves of Figure 5 and the wave-amplitude curves of Figure 4. This is not too surprising, since each depends on the same free-wave spectrum curve as a function of wave angle; wave resistance is the total area under this curve, whereas the wave amplitude is more dependent on point-wise spectral information. We are investigating further for a wide variety of ships, in research to be reported elsewhere, the quantitative correlation between wave resistance and wave amplitude.

2.5 Total drag

Although our primary aim is wave reduction, primarily on environmental grounds, there is little point in elimination of waves if the result is a vessel which is prohibitive to operate because of increased viscous drag. This would occur for example if the total surface area were increased by use of many very thin hulls. We therefore also show for each hull the total ("wave + viscous") drag in kiloNewtons as a function of speed in metres/sec. The viscous drag is estimated by the ITTC 1957 line, and no form correction is used.

Figure 6 shows the total drag of the candidate vessels. Also shown in Figure 6(a) is the total resistance curve for a monohull. Although this monohull is not strictly comparable with the multihulled vessels in the present study (primarily on the grounds of insufficient deck area) it is in any case clearly out-performed by one or other of the multihulls for speeds between 6 and 19.5ms^{-1}.

For speeds greater than 12ms^{-1}, the CAT and WEI vessels have comparable total resistance, the CAT slightly better in this range, and both

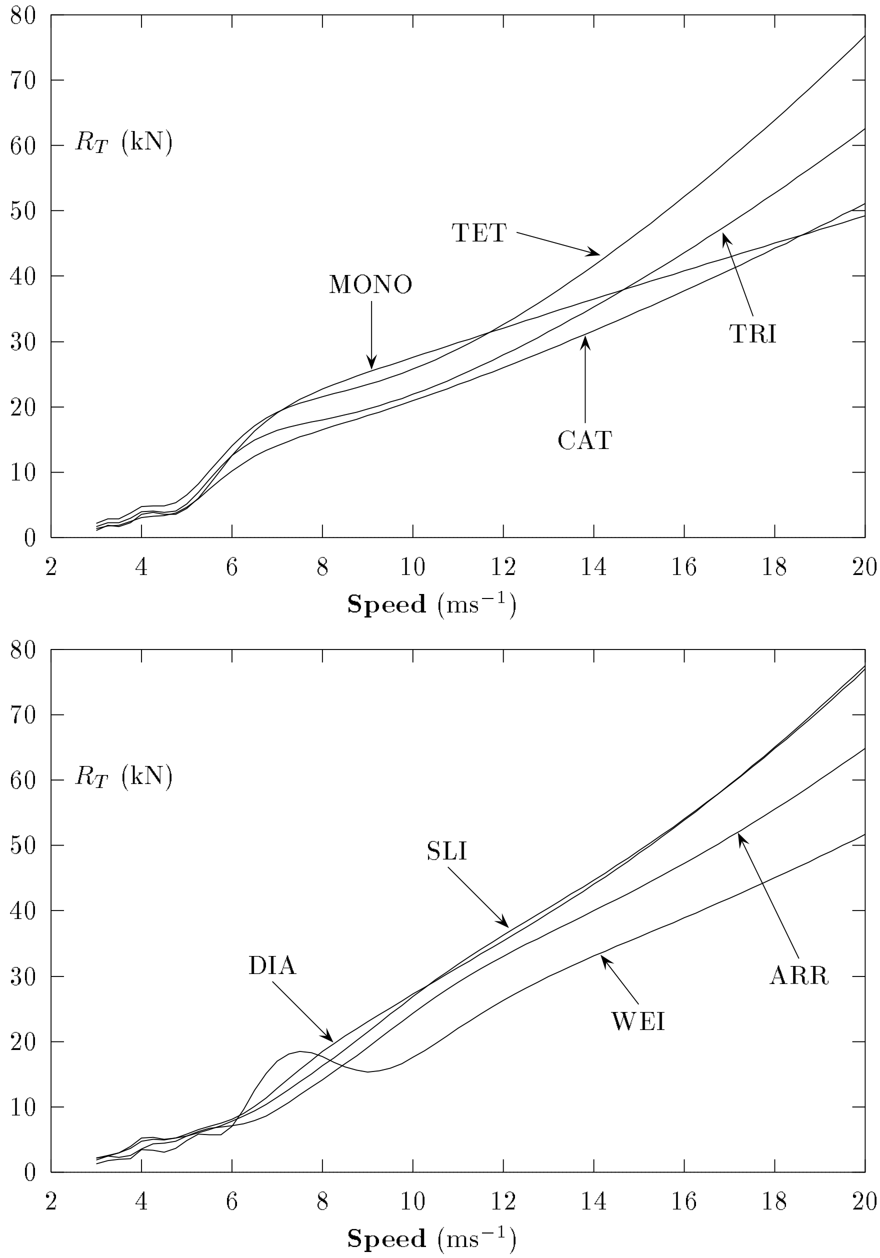

Figure 6: Total drag versus speed (31.25t).

are significantly better than the other vessels. For speeds between 8.5 and 11.5ms^{-1}, the WEI has the lowest total resistance of all the candidates, but to get into the pronounced "dip" at about 9ms^{-1}, the WEI must first overcome a hump in the total drag at about 7.5ms^{-1}. The three-hulled ARR is the best design for speeds between 6 and 8.5ms^{-1} by virtue of its ability to cancel transverse waves in this speed range. The four-hulled SLI and DIA vessels also perform well in this speed range, but at higher speeds they are (with the TET) by far the worst from a total drag viewpoint.

The general trends in the total drag shown in Figure 6 are not unexpected and the lessons to be learned are somewhat obvious; two-hulled vessels are at most speeds (from a total drag point of view) better than three-hulled vessels, which in turn are better than four-hulled vessels. This is because viscous drag is the dominant component of the total drag at most speeds, at the given (relatively-low) displacement of 31.25t. Under such circumstances, there is a large surface-area penalty which militates against use of many hulls.

2.6 Larger vessels

Since one of the objectives of ship design is to move the largest displacement at the highest speed and with the least total drag, it is natural to ask what effect an increase in displacement would have if other parameters, notably the hull length, were kept constant. To add information about larger vessels to the present example, we simply double the displacement, by doubling the beam of all hulls. This will double the wave-amplitude scale in Figure 4 and quadruple the wave-resistance scale in Figure 5. Thus the relative order of merit of the candidate vessels is maintained with respect to the first two criteria, namely minimum generated wave height and minimum wave resistance. However, since wave resistance now becomes the dominant component of the total drag for most speeds, there is a profound change in the relative merits of the candidate vessels with respect to the third criterion, namely the total drag. Importantly, the higher-multiplicity vessels now become competitive, because their extra surface area is relatively less important than it was for the smaller displacement.

Figure 7 shows the total drag of the candidate vessels, when the displacement has been doubled to 62.5t while the length, draft, and hull separation distances are all kept the same as for the 31.25t vessels. At low speeds, the staggered variants (ARR, SLI and DIA) now have significantly less total drag than all others, the four-hulled DIA being best of all by a small margin be-

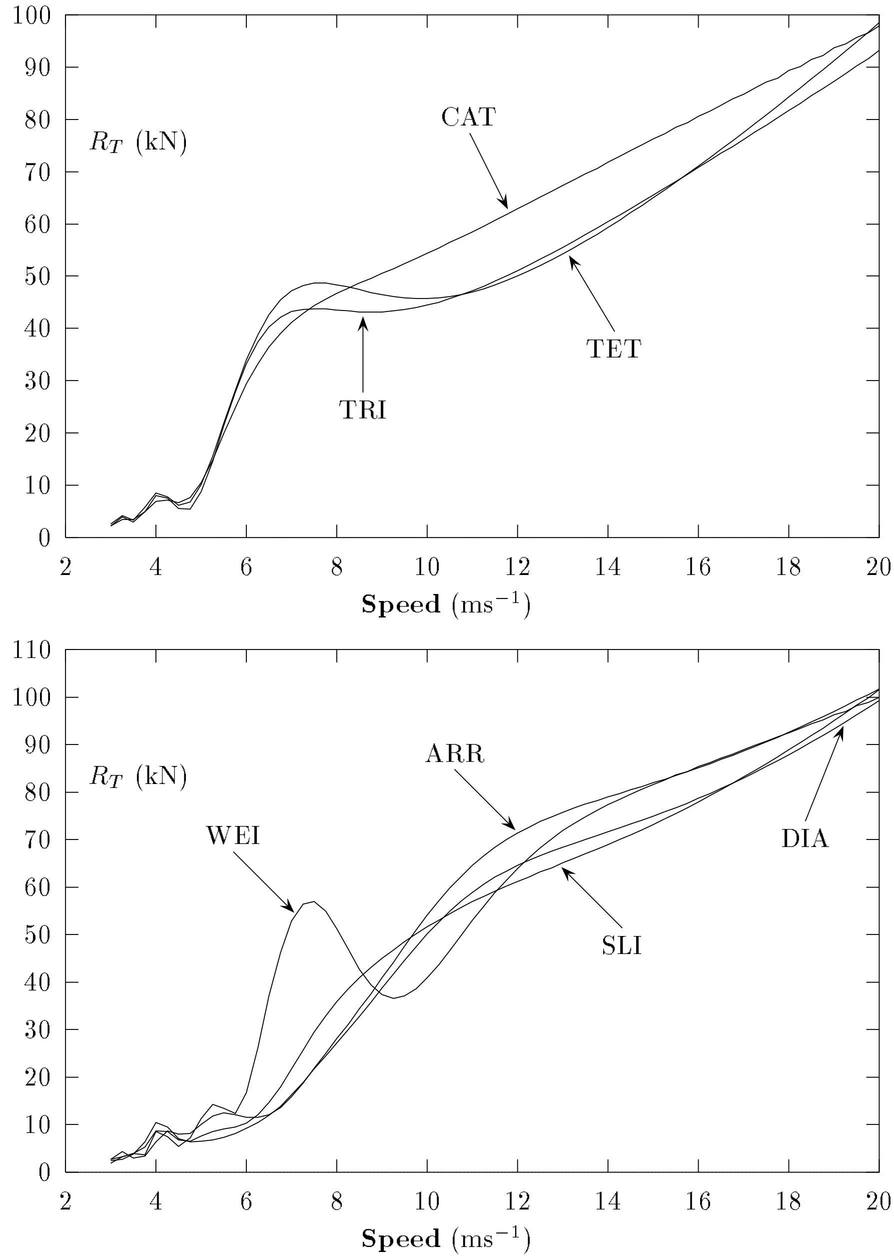

Figure 7: Total drag versus speed (62.5t).

tween 4.8 and 8.8ms^{-1}. Between 8.8 and 10.3ms^{-1}, the staggered two-hulled WEI has the lowest total drag; however outside of this narrow speed range it is uncompetitive. For speeds between 9.5 and 16.5ms^{-1}, the unstaggered three-hulled TRI and four-hulled TET vessels have similar total drag and are both superior to all other variants (except WEI in its narrow best range). For the highest speeds considered here, from 16.5 to 20ms^{-1}, the TRI vessel is now clearly best of all from the total drag point of view.

An entirely different way to generate larger vessels would be to increase the hull length. In particular, we can use Froude scaling to generalise the present results to vessels of any size, providing all lengthscales are adjusted simultaneously and identically. For example, the present 31.25t vessel can be doubled in all dimensions to produce a 250t vessel, operating at speeds $\sqrt{2} = 1.414$ times that assumed here, with twice the wave amplitude, eight times the wave resistance, and (neglecting small Reynolds-number scale effects on the skin friction coefficient) eight times the total drag. The relativities between the various candidate vessels are therefore essentially unchanged by this form of scaling.

On the other hand, if such an over-all scaling were then followed by a scaling back of the individual hull beams (by a factor of 1/8 in the above example), it would be possible to maintain a fixed displacement. This would dramatically change the relative importance of wave and viscous resistance, and hence the relative merit of the candidate vessels. The present authors [4] have used genetic algorithm techniques to minimise the total drag of multihull vessels at fixed displacement by varying the length in this way, with the not-too-surprising conclusion that very long vessels with almost no wave resistance are preferred. However, the present study is intended to suit situations where the length is constrained, such that wave resistance is of major importance and therefore needs to be reduced by careful design.

2.7 Summary of example designs

It seems that each variant considered has, at least for some range of speeds, (hydrodynamic) advantages over all or most of the other candidate vessels, so that it is important to consider the inherent disadvantages of each design. In the present example, an obvious first cut for most applications would be to reject as too unconventional and too extreme, the WEI and DIA variants, in spite of their unique low-wave characteristics at some speeds. These vessels are in any case significantly longer overall than the others, and would probably suffer in hydrodynamic comparison with other extremely long and slender designs not considered here, even perhaps with some long monohulls.

For 31.25t total displacement, the individual hulls of the four-hulled vessels are very narrow, and we may therefore have to reject these vessels from further consideration; they are in any case not competitive on a total drag basis. We are then left with CAT, TRI and ARR as the only practical choices. The CAT has a significant total drag advantage over the TRI and ARR. However it is not the best choice if lowest possible wavemaking is the dominant design criterion. In that case, ARR is the best vessel at low speeds, and TRI is superior at high speeds.

For 62.5t vessels, the beams of the individual hulls of the four-hulled vessels are more reasonable and we now have a serious choice between CAT, TRI, ARR, TET, and SLI. These candidate vessels are most naturally considered on the basis of whether we are designing for a single speed or for two (or more) design speeds. For a single design speed, the ARR and SLI designs seem to be the best choice for low speeds, and the TRI and TET are best at high speeds. The three-hulled designs are likely to be cheaper to construct than their four-hulled counterparts. However if low wave-height is the over-riding consideration on environmental grounds, then the four-hulled vessels (SLI and TET) might be preferred.

For a problem where we have two design speeds, say an economical low "cruise" speed, and a high "pursuit" speed, the CAT and SLI vessels might be good compromises. The SLI has much less low-speed drag than the CAT, and comparable total drag at high speeds, but against this are the disadvantages inherent in using four hulls.

If four hulls are acceptable, then the choice between SLI and TET depends on the speed range. If the lowest achievable wave is required at low to moderate speeds, the SLI vessel is preferable. However, the TET is a shorter, simpler design with significant drag and wave-minimising advantages at high speeds. Even its peak wave amplitude of 0.3m at $7\mathrm{ms}^{-1}$ is less than that of the CAT at the same speed, and this peak may be able to be reduced in size by careful hull design.

Much of the art in naval architecture is that of making judicious design compromises, given difficult choices and constraints. The example problem demonstrates that hydrodynamic considerations alone cannot always provide a clearly superior, practical design. They do, however, allow us to see the relative merits of the various designs, and some of their individual strengths and weaknesses.

3 Ship wave theory

3.1 Free-wave spectrum

The steady wave pattern $z = \zeta(x, y)$ of any ship, as seen at a point (x, y) sufficiently far from the ship, is of the form of a sum of plane waves travelling at various angles θ of propagation relative to the direction of motion (negative x-axis) of the ship. Thus ([3],[11],[7] p. 277)

$$\zeta(x, y) = \Re \int_{-\pi/2}^{\pi/2} A(\theta) e^{-ik(\theta)[x \cos\theta + y \sin\theta]} \, d\theta \tag{1}$$

where $A(\theta)$ is the amplitude and $k(\theta)$ the wave number of the wave travelling at angle θ. Note that the contributions to this integral from *positive* angles θ correspond to waves being propagated to the *left* or *portside* of the ship.

Once $A(\theta)$ and $k(\theta)$ are specified, we can use this result to determine the actual wave pattern, and also its total energy and hence the wave resistance

$$R = \frac{\pi}{2} \rho U^2 \int_{-\pi/2}^{\pi/2} |A(\theta)|^2 \cos^3\theta \, d\theta \tag{2}$$

of the ship. At any given ship speed U, the amplitude function $A(\theta)$ is a property only of the ship's hull geometry, whereas $k(\theta)$ is fully determined from the dispersion relation for plane waves. In infinite water depth, which we shall generally assume from now on, we have simply

$$k(\theta) = k_0 \sec^2\theta \tag{3}$$

where $k_0 = g/U^2 = k(0)$ is the wave number of pure transverse waves at $\theta = 0$.

The (complex) amplitude function $A(\theta)$, sometimes also called the free wave spectrum or Kochin function, can be computed by various means, e.g. Michell's [5] thin-ship theory, a complete nonlinear near-field computation [6],[1], or by experimental measurement [2]. For example, for a monohull with offsets $y = \pm Y(x, z)$, Michell's [5] theory indicates that

$$A(\theta) = -\frac{2i}{\pi} k_0^2 \sec^4\theta \iint Y(x, z) \exp(k_0 z \sec^2\theta + i k_0 x \sec\theta) \, dx dz. \tag{4}$$

However, in the remainder of the present paper, we shall not need to make any assumption about how this function depends on hull shape.

3.2 Multihulls

The formula (1) applies equally to multihulls as monohulls, with $A(\theta)$ the total wave amplitude function for the complete vessel. We now assume that the wave field of a multihull vessel can be constructed by linear superposition of wave fields generated by amplitude functions for each separate hull, each acting as if alone. If there are N such hulls, and the hull numbered j located at $(x, y) = (x_j, y_j)$ has wave amplitude $A_j(\theta)$, the total far-field wave is then

$$\begin{aligned}\zeta(x,y) &= \sum_{j=1}^{N} \Re \int_{-\pi/2}^{\pi/2} A_j(\theta) e^{-ik(\theta)[(x-x_j)\cos\theta + (y-y_j)\sin\theta)]}\, d\theta \\ &= \Re \int_{-\pi/2}^{\pi/2} e^{-ik(\theta)[x\cos\theta + y\sin\theta]} \sum_{j=1}^{N} A_j(\theta) e^{ik(\theta)[x_j\cos\theta + y_j\sin\theta]}\, d\theta\end{aligned}$$

That, is, the general expression (1) still applies to the whole vessel, with

$$A(\theta) = \sum_{j=1}^{N} A_j(\theta) e^{ik(\theta)[x_j\cos\theta + y_j\sin\theta]}$$

In the present paper, we suppose that all hulls have the same wave-making property, except for a constant real positive multiplier σ_j that may differ from hull to hull. If we are using the thin-ship approximation, this corresponds to hulls that are identical except that their beams vary in proportion to σ_j. Thus we set

$$A_j(\theta) = \sigma_j A_0(\theta)$$

for some given amplitude function $A_0(\theta)$. Then the combined wave amplitude is

$$A(\theta) = A_0(\theta) F(\theta)$$

where

$$F(\theta) = \sum_{j=1}^{N} \sigma_j e^{ik(\theta)[x_j\cos\theta + y_j\sin\theta]}$$

Thus $F(\theta)$ measures the interference between the hulls, and is independent of the actual wave-making property of individual hulls, as measured by $A_0(\theta)$. We can now set about minimising complex $F(\theta)$, or more usefully its real magnitude squared

$$G(\theta) = |F(\theta)|^2.$$

In doing this, we shall need to normalise the coefficients σ_j in some way, and choose to do so by setting their sum to unity, i.e.

$$\sum_{j=1}^{N} \sigma_j = 1 \ .$$

Hence each σ_j represents the fraction of the total displacement of the complete vessel that is contributed by hull j.

3.3 Independent hulls

Before attempting to choose spacings between hulls that minimise wavemaking by use of destructive interference between the wave patterns of individual hulls, it is important to clarify the situation when there is no such interference. This can be considered as the limit of large spacings for a multihull vessel. However, it can also be thought of more directly in terms of a choice between use of one large monohull and several smaller monohulls to service a route.

For example, if we split the beam of a monohull into two, and (assuming that stability and other practical considerations make the two separate thinner monohulls feasible as ships) run two such monohulls on a particular route instead of one, each thin hull will make half of the waves of the original thick hull, and have a quarter of the wave resistance. The net cost if measured by wave resistance alone is therefore halved, as is the environmental risk from wavemaking, so this is a "preferable" solution on that basis. If splitting in half is good, splitting in three is even better, etc. That is, in principle one can entirely eliminate the consequences of wavemaking by using many independent wafer-thin hulls instead of one thick hull.

The above highly-idealised scenario has been described in spite of its obvious practical flaws, because it influences the optimal hull-placement problem for realistic multihull vessels. If there is a theoretically-better arrangement with no hull-hull interaction at all, any optimisation attempted for a multihull vessel will inevitably gravitate toward infinite hull separation. Indeed, any non-trivial optimum N-hull vessel where destructive interference has been exploited must have a total wave resistance which is less than a fraction $1/N$ of the corresponding monohull resistance; otherwise its independent-hull "competitor" would have been preferred. This is not always easy to achieve.

Even when there are finite separation distances that minimise wave generation, this minimum is often very shallow, and the performance of the optimum N-hull vessel is hardly better than that of N independent hulls. In that case, a more realistic question to ask is, what is the minimum separation distance at which a performance close to (say with a wave resistance not more than 10% higher than) the independent-hull limit can be achieved? This is essentially a "coupled" optimisation matter, arising only when we combine the hull-hull interference factor $G(\theta)$ with the spectrum $|A_0(\theta)|^2$ for a particular hull shape. For the present study, we ignore such coupling, and seek simply to minimise $G(\theta)$.

4 Analysis of optimal spacing

Let us now consider the task of minimising $G(\theta)$ for increasing values of N. Clearly there is nothing further to be said about the case $N = 1$, with $G(\theta) = \sigma_1^2 = 1$. Minimisation of wavemaking for monohulls is (naturally!) a matter of choosing the wave amplitude $A_0(\theta)$ of the individual hull, and we are not considering that choice here.

We consider each value of N in turn. A relatively exhaustive treatment is attempted in order to avoid missing any promising configurations, but some mathematical details are diverted to Appendices.

4.1 Di-hulls

For a two-hulled vessel or catamaran with $N = 2$ we have

$$G(\theta) = \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2 \cos\left[k(\theta)(s\cos\theta + w\sin\theta)\right]$$

Here $s = x_1 - x_2$ is the longitudinal separation or stagger, and $w = y_1 - y_2$ is the lateral separation or width. Clearly G lies between a minimum of $(\sigma_1 - \sigma_2)^2 = 1 - 4\sigma_1\sigma_2$ and a maximum of $(\sigma_1 + \sigma_2)^2 = 1$, irrespective of θ. Under those circumstances, the best choice must be $\sigma_1 = \sigma_2 = 1/2$, allowing

$$G(\theta) = \cos^2\left[\frac{1}{2}k(\theta)(s\cos\theta + w\sin\theta)\right]$$

to vanish at its minimum. That is, the best wave-cancelling di-hull always has two identical hulls.

This is true whether the di-hull is conventionally symmetric, i.e. has side-by-side identical hulls with $s = 0$, or is unsymmetric, with nonzero stagger s. First let us consider conventional catamarans with $s = 0$. The only parameter now available for minimisation is the width w. However, at $\theta = 0$ we have $G(0) = 1$ irrespective of w. That is, there is nothing we can do to reduce pure transverse waves by varying the width of symmetric di-hulls, a physically obvious result.

Otherwise, our aim must be to minimise G at some non-zero angles of interest. Clearly we can entirely eliminate waves at any particular angle $\theta = \theta_0$ by setting

$$k(\theta_0)w \sin\theta_0 = \pi, 3\pi, 5\pi, \ldots$$

If we select a particular angle θ_0 as the lowest at which there will be no waves, then we must choose the width as

$$w = \frac{\pi}{k(\theta_0)\sin\theta_0}$$

Then $G(\theta)$ will simply oscillate infinitely many times between 0 and 1, taking the value 0 when θ satisfies

$$\frac{k(\theta)\sin\theta}{k(\theta_0)\sin\theta_0} = 1, 3, 5, \ldots$$

and the value 1 when θ satisfies

$$\frac{k(\theta)\sin\theta}{k(\theta_0)\sin\theta_0} = 2, 4, 6, \ldots$$

For example, to entirely eliminate waves travelling at $\theta_0 = 60°$ in infinite water depth, we must choose a width w satisfying $k_0 w = \pi/(2\sqrt{3}) = 0.907$, or a width/shiplength ratio of $0.907F^2$, where F is the Froude number. The function $G(\theta)$ then possesses a parabola-like zero minimum at $\theta = 60°$, remaining below 0.25 in the 10-degree range $53.8° < \theta < 63.8°$. Hence this configuration will be effective in reducing waves whenever the base hull has a spectrum with a main maximum near $\theta = 60°$ which is largely confined to such a range of θ values. This seems to be the case for most conventional hulls at Froude numbers of about $F = 0.6$, and the required width to eliminate waves at $\theta = 60°$ is then about one-third of the shiplength.

This is illustrated in Figure 8, which gives the free-wave spectrum, $dR/d\theta$ for vessels similar to the family used in Section 2, at a speed of 8.25ms^{-1}. Here

R is the actual wave resistance in kiloNewtons, and $dR/d\theta$ is the integrand in the formula for the wave resistance, so that the resistance is the area under each of the curves. Thus $dR/d\theta$ is a measure of the energy in the component of the ship wave pattern that is propagating at direction θ. The speed was chosen as 8.25ms^{-1} so that the peak in the free-wave spectrum for the monohull occurs at $\theta = 60°$.

The present theory shows that we can completely eliminate waves propagating at $\theta = 60°$ for a catamaran by choosing the lateral spacing between hulls as 6.293m. Although Figure 8 confirms this result, dramatic cancellation is apparent only for a small range of θ.

As indicated in Section 3.3, the best width to minimise total wavemaking or wave resistance may well be infinite. That is, total elimination of waves at some particular angle θ is not necessarily optimal for a particular hull shape and spectrum.

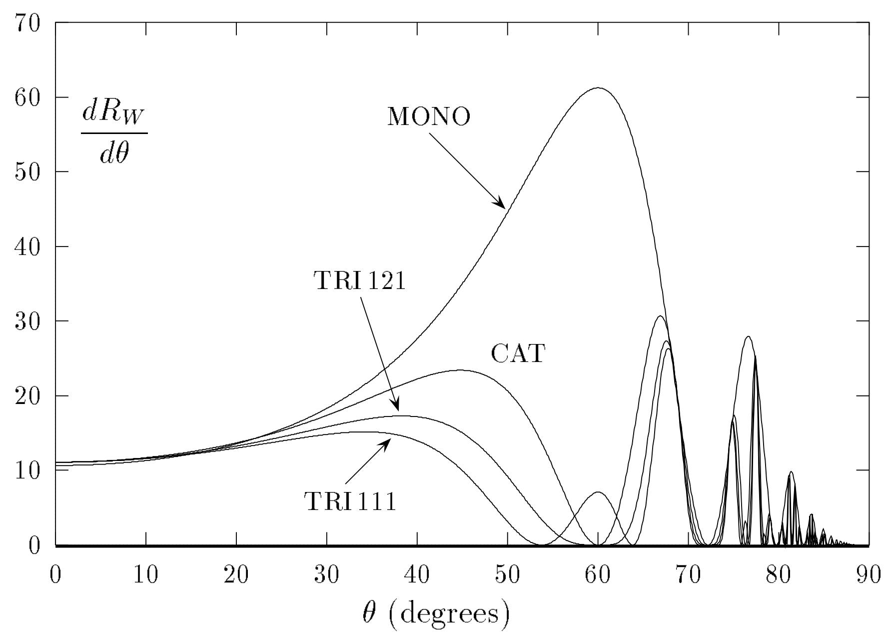

Figure 8: Free wave spectra for various multihulls at 8.25ms^{-1}.

4.2 Laterally-unsymmetrical di-hulls

Let us now turn to unsymmetrically staggered di-hulls, or "Weinblums" [10] with $s > 0$. It is now possible to cancel the pure transverse waves at $\theta = 0$, by choosing the longitudinal stagger as $s = \pi/k_0$. Any odd multiple of that quantity also eliminates transverse waves, but let us assume the smallest stagger is used. This simply corresponds to placing the second hull exactly one half-wavelength behind the first, with their equal-magnitude transverse waves exactly out of phase.

All that is left to do is to choose an optimal value for the lateral separation w. At first sight this is no more encouraging than for symmetric di-hulls, since the function $G(\theta)$ still oscillates infinitely many times between 0 and 1. Indeed, for $\theta > 0$ (waves on the port side), this vessel has no advantage over conventional catamarans with respect to cancellation of diverging waves. However, when $\theta < 0$, it is possible to choose a lateral separation such that $G(\theta)$ remains close to zero over a remarkably large range, and we give the detailed mathematics of this choice in Appendix 1.

The result shows a rather broad minimum at a lateral separation of $w = 0.88U^2/g$, and is such that $G < 0.005$ for $-64.5° < \theta < +7.3°$. This lateral separation corresponds (in combination with the longitudinal separation of $s = 3.14U^2/g$) to the line joining centres of the two hulls making an angle of 15.7° to the direction of motion, somewhat inside the Kelvin angle 19.5° of the wave pattern of the leading hull. Hulls so placed will make very small waves on the starboard side. An alignment of the trailing hull to starboard not far from a line at the Kelvin angle is intuitively reasonable as a means to cancel the waves propagated along that line by the leading hull.

4.3 Tri-hulls

If $N = 3$, the general expression for the (complex) interference factor is

$$F(\theta) = 1 - \sigma_2 - \sigma_3 + \sigma_2 e^{ik(\theta)[x_2 \cos\theta + y_2 \sin\theta]} + \sigma_3 e^{ik(\theta)[x_3 \cos\theta + y_3 \sin\theta]}$$

where we have set $x_1 = y_1 = 0$ without loss of generality, i.e. placed hull 1 at the origin, and used the scaling law to eliminate σ_1.

We consider here only laterally symmetric vessels, assuming that $\sigma_2 = \sigma_3 = \sigma$, $x_2 = x_3 = s$ and $-y_3 = y_2 = w$. That is, the vessel consists of a main hull of displacement fraction $1-2\sigma$, with symmetrically placed identical side hulls each of displacement fraction σ at a stagger distance s behind the main hull, one a distance w to port and one w to starboard of the main hull.

Then

$$F(\theta) = 1 - 2\sigma + 2\sigma e^{iks\cos\theta} \cos(kw \sin\theta)$$

with magnitude squared

$$G(\theta) = (1-2\sigma)^2 + 4\sigma(1-2\sigma)\cos(ks\cos\theta)\cos(kw\sin\theta) + 4\sigma^2\cos^2(kw\sin\theta)$$

We hope to use longitudinal stagger s to set $G(0) = 0$. However, if there is no stagger, i.e. $s = 0$, then $G(0) = 1$ for all w, and (as with conventional catamarans) we cannot eliminate pure transverse waves. Let us first consider that case (of conventional so-called "trimarans") in detail.

4.4 Side-by-side tri-hulls

Now if $s = 0$ we have

$$F(\theta) = 1 - 4\sigma\sin^2(\frac{1}{2}kw\sin\theta)$$

which varies between maxima of 1 and minima of $1 - 4\sigma$. An obviously favourable choice is $\sigma = 1/4$, which forces that minimum to be zero, with

$$F(\theta) = \cos^2(\frac{1}{2}kw\sin\theta).$$

This is the case where the outrigger hulls are each half of the displacement of the central hull. It produces results comparable to and somewhat better than that of a conventional catamaran.

Indeed, if we assume the same between-hull spacing w in each case, so that the trimaran is twice as wide as the catamaran, then $F(\theta)$ for the $\sigma = 1/4$ trimaran is identical to $G(\theta)$ for the catamaran. That is, the interference factor for this trimaran is the *square* of that for the catamaran. This is clearly favourable, producing wider minima (of value 0) and narrower maxima (of value 1). Any range of angles θ where the catamaran has $G(\theta) < 1/4$ (for example, the 10-degree range $53.8° < \theta < 63.8°$ for $\theta_0 = 60°$ quoted above) will be such that $G(\theta) < 1/16$ for the trimaran, and hence will lead to better wave reduction when the base hull spectrum is peaked in that range.

There is some potential gain from using σ values somewhat greater than 1/4, i.e. from allowing each outrigger displacement to be more than half of that of the central hull. In that case, F can be allowed to take a (small) negative value $1 - 4\sigma$ at its minimum, with $G = F^2$ therefore taking a small positive local maximum closely bracketed with zeros. This can produce an even wider range of θ values where G takes relatively small values. For example, if $\sigma = 0.3125$ (compared to the central hull $1 - 2\sigma = 0.375$), the above range of angles where $G(\theta) < 1/16$ at $\theta_0 = 60°$ extends from 10 degrees to nearly 14 degrees, namely $50.9° < \theta < 64.8°$. Even the case of three identical hulls ($\sigma = 1/3$) is worth considering from this point of view, as this gives a minimum in F of $-1/3$ or a local maximum in G of 1/9. The case of three identical hulls is especially significant when there is no hull interference, i.e. in the large-separation limit, as discussed in Section 3.3.

Figure 8 as already discussed for catamarans also shows some trimaran free-wave spectra illustrating the theoretical considerations of the present section. The "TRI111" vessel has three identical hulls and is similar to (but with a different lateral separation) the vessel called TRI in Section 2. The vessel "TRI121" has a central hull with twice the displacement of one of the outrigger hulls, as required by the above theoretical optimum.

The present theory shows that we can completely eliminate waves propagating at $\theta = 60°$ for both the catamaran and the TRI121 vessels by choosing the lateral spacing between hulls as 6.293m. For the TRI121 vessel, the region of cancellation is over a much wider range of angles than for the catamaran. The TRI111 arrangement does not eliminate completely the waves propagating at $\theta = 60°$, but produces an even wider range of angles where the energy is relatively low. Indeed, the TRI111 has the lowest wave resistance (area under the curves) of the four vessels illustrated in Figure 8. This is achieved primarily by reducing waves propagating between $\theta = 20°$ and $\theta = 53°$, the drag saving in this region more than compensating for the worse performance very close to $\theta = 60°$.

The type of low-wave configuration discussed in the present section involves cancellation of diverging waves, with no serious attempt being made to cancel transverse waves. It is therefore mainly useful at relatively high speeds, typically hull-length Froude numbers $F > 0.55$ (speeds greater than 7.5m, for the example 19.1m hull), when the base hull makes little transverse waves. Cancellation of transverse waves can only be achieved by use of a significant amount of longitudinal stagger, as treated in the following section. We have however also investigated the possibility that the present (diverging wave) cancellation could be enhanced by use of a (small) longitudinal stagger, but have reached a negative conclusion on that matter. That is, $s = 0$ is a local minimum in G, for configurations which eliminate waves at any non-zero angle $\theta = \theta_0$; longitudinal stagger is only of value if it is large enough to be comparable to the transverse half-wavelength, and is otherwise detrimental.

4.5 Arrow-shaped tri-hulls

As with the Weinblum di-hull, it is possible to carry out a complete mathematical determination of the optimum layout of a three-hulled vessel in a laterally symmetric arrow-shaped configuration with $s > 0$, and this is provided in Appendix 2.

In summary, the central hull should have half the total displacement, i.e. have twice the beam of each of the identical side hulls. The longitudinal stagger should as usual be $s = \pi U^2/g$ to eliminate transverse waves. Our final task is then to choose the lateral separation w, or equivalently the half-angle of the "arrow".

Again there is a rather broad minimum in G as a function of w, and a good compromise is $w = 0.85U^2/g$, corresponding to an arrow half-angle of $15.1°$. This angle for the line joining hull centres places the outrigger hulls inside the Kelvin angle of the central hull, and is slightly smaller than the corresponding angle for the Weinblum di-hull. For that choice, we find that $G < 0.26$ for all θ in the range $|\theta| < 71.5°$. This is nowhere near as good a cancellation as is achieved for the Weinblum di-hull on its starboard side, but this laterally symmetric tri-hull vessel cancels waves on both sides, and the range of angles over which such cancellation occurs is quite impressive.

4.6 Side-by-side tetra-hulls and higher

It is not difficult to repeat the di-hull and tri-hull analysis for side-by-side multihulls of any multiplicity N. For configurations entirely eliminating waves at a given angle $\theta = \theta_0$, the interaction factor G can be made to take the value

$$G(\theta) = \left[\cos(\frac{w}{2}k(\theta)\sin\theta)\right]^{2N-2}$$

by placing all hulls in an equally-spaced side-by-side array, the lateral distance between hulls being

$$w = \frac{\pi}{k(\theta_0)\sin\theta_0}.$$

Thus the total width $(N-1)w$ of the vessel increases with multiplicity, in proportion to $N-1$. The individual displacements must vary in accordance with Pascal's triangle, i.e. the j'th hull has a fraction

$$\sigma_j = \left(\frac{1}{2}\right)^{N-1}\binom{N-1}{j-1}$$

of the total, expressed in terms of binomial coefficients. In particular, each outermost hull has a rapidly diminishing fraction $\sigma_1 = \sigma_N = (1/2)^{N-1}$ of the total displacement.

For example, the optimum tetra-hull ($N = 4$) has two central hulls each with 3/8 of the displacement, and two outer hulls each with 1/8 of the displacement. Similarly, the optimum penta-hull ($N = 5$) has a central hull with 3/8 of the displacement, two intermediate hulls each with 1/4 of the displacement and two outer hulls each with 1/16 of the displacement.

In the limit as $N \to \infty$ there is zero wave resistance, irrespective of the shape of the individual hulls, since the interference factor $G(\theta)$ approaches zero for all θ. However, this is an infinitely wide array, with all hulls of vanishingly small beam. Any such array will have zero wave resistance, as discussed in Section 3.3. Beyond about $N = 5$, the present array with extremely thin outriggers is hardly practical.

The above results are "optimal" only in the sense of minimising wave-making near a particular angle $\theta = \theta_0$. In practice, the independent-hull limit of large spacings again often provides the least wave resistance, and this corresponds to N identical hulls rather than the "Pascal" distribution suggested here.

4.7 Arrow-shaped tetra-hulls

A special case of the tetra-hull with $N = 4$ is comparable to the arrow-shaped tri-hull, namely a laterally-symmetric configuration with two leading hulls a small distance $2v$ apart and two trailing hulls a greater distance $2w > 2v$ apart, with the trailing hulls staggered s relative to the leading hulls. This type of vessel has been called a "Slice" [8]. It is easy to see that for transverse wave cancellation we must have all four hulls identical, and $s = \pi/k_0$ as before. In effect the present configuration is obtained by "slicing" the leading hull of the arrow tri-hull in two, and then shifting these two halves apart laterally by $2v$. Since we now have an extra parameter v, there is some hope of achieving even better wave cancellation.

Appendix 3 gives a summary of the mathematical optimisation problem for this vessel, although no attempt is made to provide as exact an optimisation as for the previous cases. The conclusion is that the "best" separations are approximately $v = 0.41U^2/g$ and $w = 1.26U^2/g$. These choices give $G < 0.22$ for $|\theta| < 71°$. This is a 14% improvement over the arrow tri-hull, which is useful but not spectacular, and the extra complexity and surface area of this vessel may not always be warranted. Note that the angle of the line between centres of the hulls on each side of this vessel is approximately the same 15° as for the arrow tri-hull.

4.8 Diamond-shaped tetra-hulls

The arrow-shaped tetramaran is not the only laterally-symmetric possibility with $N = 4$. Suppose that the four hulls are in a diamond-shaped array, consisting of two identical centerline hulls of displacement $\sigma_1 = \sigma_2 = 1 - 2\sigma$ in tandem with their centres a longitudinal distance $2s$ apart, and two identical outrigger hulls with displacement $\sigma_3 = \sigma_4 = \sigma$ side-by-side midships with their centreplanes a lateral distance $2w$ apart. That is, $(x_1, y_1) = (-s, 0), (x_2, y_2) = (+s, 0), (x_3, y_3) = (0, +w)$ and $(x_4, y_4) = (0, -w)$. Then

$$F(\theta) = (1 - 2\sigma)\cos(ks\cos\theta) + 2\sigma\cos(kw\sin\theta)$$

Cancellation of transverse waves $F(0) = 0$ demands

$$\sigma = -0.5\frac{\cos(k_0 s)}{1 - \cos(k_0 s)}$$

For example, all four hulls are identical with $\sigma = 0.25$ if $\cos(k_0 s) = -1$, the shortest vessel of which has $s = \pi/k_0$ as before. Note however, that the total longitudinal stagger $2s$ is then twice that of the previous vessels. Otherwise, we require s to lie in the range between $0.5\pi/k_0$ and $1.5\pi/k_0$, with the outriggers thinner than the central hulls.

Trial and error is the only reasonable procedure for minimisation of $F(\theta)$ or $G(\theta) = F^2$ with respect to the spacing parameters s, w in the present case, using our experience with simpler vessels as a guide. The results can be rather spectacular. For example, in the identical-hull case $s = \pi/k_0$, it is possible by choosing $w = .290s$ (diamond apex half-angle 16.2°) to achieve $G < 0.001$ for all $\theta < 62.5°$! Although this is an extreme degree of wave cancellation, it is perhaps overkill, and we can do "better" in terms of the range of θ values where cancellation occurs, by using a narrower apex angle, at the expense of a somewhat greater G. For example, if $w = 0.265s$ (half-angle 14.8°), we have $G < 0.013$ for all $\theta < 71°$, and this is still a very impressive amount of wave cancellation. The choice between narrow and wide diamonds then depends on the amplitude function $A_0(\theta)$ of the individual hulls; if there is significant energy in the range $62.5° < \theta < 71°$ we should use the narrow diamond, but if not, there may be some advantage from the greater low-θ effectiveness of the wider diamond.

There is not a lot to be gained by use of different values of the stagger s; none at all from longer vessels. Slightly shorter vessels with smaller outriggers do give a small improvement, but mostly in the sense of extending the range of θ. For example, if $s = 0.86\pi/k_0$, the best width is $w = 0.22s$ (half-angle $12.4°$), and this vessel has the same wave cancellation $G < 0.013$ as the narrow identical-hull vessel, but over a slightly greater angle range $\theta < 73°$. This vessel will be about 10% shorter overall than the identical-hull vessel, with a 10% reduction in the outrigger thickness relative to the central hulls.

5 Conclusion

In order to choose optimal hull placements for multihulls, we have here used the linear superposition principle and direct proportionality of the generated wave amplitude to hull beam. These are inherent properties of Michell's [5] thin-ship theory. However, we have not had to make use of Michell's explicit relationship (4) between hull shape and free-wave spectrum. Indeed, apart from the specific design example for Wigley hulls in Section 2, we have not used any information about hull shape. That example is potentially misleading in some ways, as there is no reason to believe that Wigley hulls are in any sense optimal for minimim wave-making purposes. Further work is needed to clarify relations between hull shape and hull placement.

In this, the centenary year of the publication of Michell's 1898 landmark paper, his legacy can still be of great use in qualitative ship design.

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Appendix 1: Optimisation of Weinblum di-hull

Our mathematical task is minimisation over the largest possible range of angles θ, by choice of the longitudinal and lateral separations s, w, of the function

$$G(\theta) = \cos^2\left[\frac{\pi}{2} f(\theta)\right]$$

where

$$\pi f(\theta) = k(\theta)\left[s\cos\theta + w\sin\theta\right].$$

We assume that the wave number $k(\theta)$ is a positive symmetric function of θ with the property $k \to +\infty$ as $|\theta| \to 90°$. When $w = 0$ (tandem hulls), we have $f \geq 0$, and G again oscillates between 0 and 1 infinitely many times as $|\theta| \to 90°$ and $f \to +\infty$. However, when $w > 0$, there is no longer symmetry of f with respect to θ, and although the behaviour for $\theta > 0$ is similar to that at $w = 0$, some interesting new things happen for $\theta < 0$. In particular, now $f \to -\infty$ as $\theta \to -90°$, but not necessarily monotonically. Clearly if w is small, there will be a tendency to follow the $w = 0$ trend toward large positive values of f until θ is close to $-90°$, when f will plunge to large negative values.

At this stage, to make further progress it is necessary to specify the form of the dispersion relation $k(\theta)$, and we use the infinite-depth relation (3). We also assume that the longitudinal stagger s is chosen as the lowest value which exactly eliminates all transverse waves by requiring $f(0) = 1$, namely $s = \pi/k_0$. Then

$$f(\theta) = \sec\theta + W\sec^2\theta\sin\theta$$

where $W = k_0 w/\pi$. Figure 9 shows $f(\theta)$ for various W.

Our task can now be reduced to that of attempting by choice of W to keep $f(\theta)$ as close as possible to 1. Now we find that there is a minimum in f at $\theta = \theta_{\min} < 0$ satisfying $f(\theta_{\min}) < 1$ where

$$-\tan\theta_{\min} = \frac{1 - \sqrt{1 - 8W^2}}{4W}$$

and a maximum $f(\theta_{\max}) > 1$ where

$$-\tan\theta_{\max} = \frac{1 + \sqrt{1 - 8W^2}}{4W}$$

(we assume $W^2 < 1/8$). Thus as θ decreases from 0, f first decreases from 1 to its minimum at $\theta = \theta_{\min}$, then passes back above 1 till it reaches its

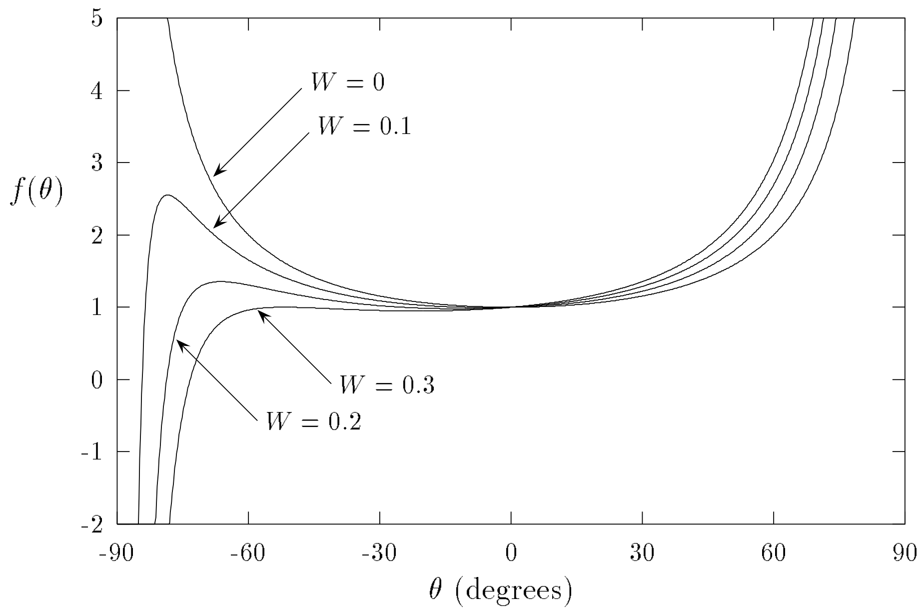

Figure 9: Phase function $f(\theta)$ for the Weinblum di-hull.

maximum at $\theta = \theta_{\max}$. After that, f again decreases below 1, but then continues to decrease monotonically toward $-\infty$ as $\theta \to -90°$. It is during the latter monotonic decrease that $G(\theta) = \cos^2(\pi f/2)$ will oscillate between 0 and 1.

However, before then, f negotiates its own minimum and maximum, and we can ensure by choice of W that these are both close to 1, and hence G is close to zero. In fact, the best choice of W is that which causes the minimum of f to lie exactly as far below 1 as the maximum does above 1, i.e.

$$f(\theta_{\max}) - 1 = 1 - f(\theta_{\min}).$$

After some manipulation, it can be shown that this occurs when W^2 satisfies the cubic equation

$$512(W^2)^3 + 1728(W^2)^2 - 152W^2 + 1 = 0$$

which has the solution $W = 0.280674$ in the required range. With this choice of W, we have $f(\theta_{\min}) = 0.95540$ occurring at $\theta_{\min} = -19.243°$, and $f(\theta_{\max}) = 1.04460$ occurring at $\theta_{\max} = -55.079°$, i.e. the maximum excursion in $f(\theta)$ from 1 is of magnitude 0.04460. The resulting local maximum value of $G(\theta)$ is 0.00490, taken equally at $\theta = -19.243°$ and $\theta = -55.079°$. As θ decreases further from $-55.079°$, G goes to zero at $\theta = -62.4°$, then increases, passing through 0.00490 again at $\theta = -64.5°$ before reaching its first global maximum of 1 at $\theta = -74.3°$. This is then followed by infinitely many swings between 0 and 1 as $\theta \to -90°$.

Meanwhile, when $\theta > 0$, the behaviour of $G(\theta)$ is more conventional, and differs little from the case $w = 0$. G passes through 0.00490 at $\theta = +7.3°$, with its first global maximum value of 1 occurring at $\theta = +48.7°$. The waves on the port side of this vessel are not minimised, although the absence of transverse waves is still favourable.

In summary, providing we choose $W = 0.280674$, or $w = 0.88176U^2/g$, the interference function $G(\theta)$ is less than 0.00490 over the remarkably large range $-64.5° < \theta < +7.3°$. The line joining centres of the two hulls makes an angle of $\arctan(0.280674) = 15.678°$ to the direction of motion.

Although we have been able to compute this particular optimum to at least 5-figure accuracy, $G = 0.00490$ is a rather broad minimum around $W = 0.280674$, and in practice even as much as 10% variations about the true optimum still give excellent results. Wider configurations retain good cancellation, but over a narrower range, e.g. $W = 0.30$ (centerline angle 16.7°) gives a maximum $G = 0.0067$ for $\theta > -61.0°$. On the other hand, narrower configurations degrade the extreme minimum value more quickly, but retain a reasonably low G for larger $|\theta|$ values, e.g. $W = 0.27$ (centreline angle 15.1°) gives a maximum of $G = 0.0131$ for $\theta > -65.7°$. The best choice then depends on whether the individual hull spectrum $A_0(\theta)$ has significant energy at $|\theta|$ values in the 61 to 66 degree range.

Appendix 2: Optimisation of arrow tri-hull

Our mathematical task for this vessel is minimisation over the largest possible range of angles θ, by choice of the individual outrigger displacement fraction σ, and the longitudinal and lateral separations s, w, of the function

$$G(\theta) = (1-2\sigma)^2 + 4\sigma(1-2\sigma)\cos(ks\cos\theta)\cos(kw\sin\theta) + 4\sigma^2\cos^2(kw\sin\theta)$$

In particular, at $\theta = 0$,

$$G(0) = 1 - 8\sigma(1-2\sigma)\sin^2\left(\frac{1}{2}k_0 s\right).$$

If $s > 0$, $G(0)$ varies from a maximum of 1 when the sine vanishes, to a minimum of $(1-4\sigma)^2$ when the sine has magnitude 1, i.e. when $k_0 s/\pi = 1, 3, 5, \ldots$. That minimum value can be made to be zero if $\sigma = 1/4$. That is, as is physically obvious, pure transverse waves can be eliminated entirely if the main hull has half the total displacement and the side hulls each have a quarter the total displacement, with the side hulls staggered longitudinally by an odd multiple of a half wavelength. This means that the side hulls produce transverse waves that are (in total) of the same amplitude as those made by the main hull, but 180° out of phase with them. Note that this conclusion holds for any value of the lateral half-separation w. We thus assume from now on that $\sigma = 1/4$ and that s takes the minimum value π/k_0 needed to cancel transverse waves.

All that is left to do is to choose the lateral spacing w, to minimise the residual interference factor

$$G(\theta) = \frac{1}{4} + \frac{1}{2}\cos(ks\cos\theta)\cos(kw\sin\theta) + \frac{1}{4}\cos^2(kw\sin\theta)$$

with $s = \pi/k_0$. Again, let us now assume the infinite-depth dispersion relation (3). Then

$$G(\theta) = \frac{1}{4} + \frac{1}{4}C^2 + \frac{1}{2}C\cos(\pi\sec\theta)$$

where

$$C = \cos(\pi W \sec^2\theta\sin\theta)$$

and $\pi W = k_0 w$.

Clearly G is bounded by 1, which would occur if simultaneously C and $\cos(\pi\sec\theta)$ take the values $+1$ or -1. Now we have already assured that $G(0) = 0$, so as we increase θ from zero, $G(\theta)$ rises to its first maximum at some angle $\theta = \theta_0$. One possible aim would be to minimise the height of that maximum by choice of W. In fact it is not hard, by setting to zero both $dG/d\theta$ and dG/dW, to show that this demands that both $C = 0$ and $\cos(\pi\sec\theta) = 0$. Hence the least possible value of the first maximum of the interference factor $G(\theta)$ is exactly $1/4$, and this occurs first when $\sec\theta = 3/2$, i.e. at $\theta = \theta_0 = \arccos(2/3) = 48.20°$. Meanwhile, for C to vanish at that angle, we must have $\pi W\sec^2\theta\sin\theta = \pi/2$, or

$$W = \frac{1}{2}\cos^2\theta_0/\sin\theta_0 = \frac{2}{3\sqrt{5}} = 0.29814$$

When W takes that value, after reaching the first peak of 0.25 at $\theta = 48.2°$, $G(\theta)$ falls to its first minimum at about $\theta = 60°$, the magnitude of which is very small (less then 0.00001), then rises to a second maximum of 0.344 at $\theta = 66.6°$, then another minimum, another maximum, etc. The reason for the very small magnitude of the first minimum can be seen by noting that it is also possible to choose another W so that this minimum is *exactly* zero, if $C = -1$ and $\cos(\pi \sec\theta) = +1$. This happens at exactly $\theta = 60°$ when $W = 1/(2\sqrt{3}) = 0.28868$, which is close to, a little below, the above optimum value.

In fact, it is somewhat advantageous to decrease W even a little more. For example, suppose we demand that the *second* maximum of G be exactly 1/4, rather than the first. The second angle where $\cos(\pi \sec\theta) = 0$ is $\theta = \arccos(2/5) = 66.42°$, and then $C = 0$ if $W = 6/(5\sqrt{21}) = 0.26186$. With this choice of W, the first peak (at about $\theta = 48.6°$, close to the previous 48.2°) is slightly elevated, of magnitude about 0.259 rather than the optimum 0.25, but this (less than 4%) increase in the size of the first peak may under some circumstances be warranted because of the more substantial reduction in the size of the second peak, from 0.344 at $\theta = 66.6°$, to 0.25 at $\theta = 66.4°$.

A good compromise is $W = 0.270$ which gives approximately the same size for both peaks, namely $G = 0.256$. As we move to even higher angles, $G(\theta)$ remains below 0.256 until $\theta = 71.5°$. Then as θ increases beyond 71.5° toward 90°, there are more and more closely spaced maxima and minima in G, with some maxima approaching the global maximum of 1. This compromise optimum has an apex half-angle of $\arctan(0.27) = 15.1°$.

In summary, good wave reduction by a factor of about a quarter in the energy spectrum for all $\theta < 71°$ occurs for W values between about 0.26 and 0.29, or trimaran half-angles between about 14.5° and 16°. Again it is interesting to note that the optimum trimaran fits entirely inside the Kelvin angle of the leading hull.

Appendix 3: Optimisation of arrow tetra-hull

Our mathematical task for this vessel is minimisation over the largest possible range of angles θ, by choice of the two lateral separations v, w, of the function

$$G(\theta) = \frac{1}{4}\cos^2(kv\sin\theta) + \frac{1}{2}\cos(ks\cos\theta)\cos(kv\sin\theta)\cos(kw\sin\theta) + \frac{1}{4}\cos^2(kw\sin\theta)$$

having demanded that the hulls be identical, with longitudinal separation $s = \pi/k_0$.

Again for infinite depth, we can write

$$G(\theta) = \frac{1}{4}B^2 + \frac{1}{4}C^2 + \frac{1}{2}BC\cos(\pi\sec\theta)$$

where

$$B = \cos(\pi V \sec^2\theta\sin\theta)$$

and $\pi V = k_0 v$ and C is defined as before.

Although it is no longer reasonable to seek such accurate optima as before, we can be guided by our experience with simpler configurations. In particular, as with the arrow tri-hull (which is the case $V = 0$ or $B = 1$), we can for any V choose W so that the first two peaks of G have the same size. Then we increase V slowly from zero until that peak size is minimal. The conclusion is that the best choices are $V = 0.13, W = 0.40$, with the size of the first two peaks in G being 0.22.