AIRFOIL NOSE SHAPES DELAYING LEADING-EDGE SEPARATION

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ABSTRACT

We show that smooth local modifications to a parabolic leading edge can delay separation of the laminar boundary layer on the upper surface of an airfoil. Symmetric modifications, of the nature sharpening the nose allow to achieve an increase of up to 8% in the unseparated angle of attack. Further improvements are possible (at least 11% increase) for unsymmetrical (drooped) noses.

Present consideration adopts assumptions of the thin-airfoil theory and was motivated by the work carried out in

Van Dyke, M.D. (1954) Subsonic edges in thin-wing and slender-body theory. *NACA* TM 3843.

Werle, M.J. and Davis, R.T. (1972) Incompressible laminary boundary layers on a parabola at angle of attack. A study of the separation point. *ASME J. Appl. Mech.*, 7-12.

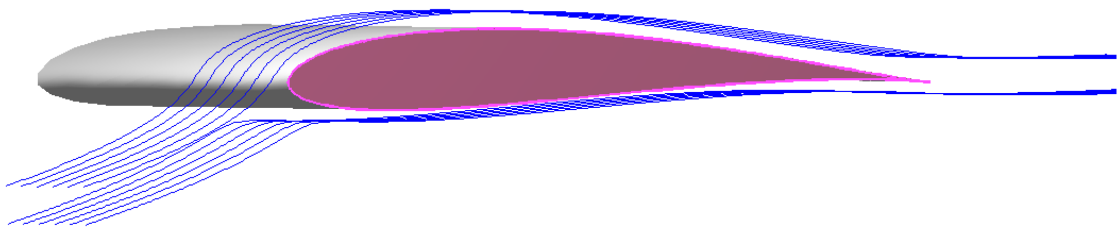
Ruban, A.I. (1981) Asymptotic theory of short separation regions on the leading edge of a slender airfoil. *Izv. Akad. Nauk SSSR Mech. Zhidk. Gaza*, **1**, 42-51.

Tuck, E.O. (1991) A criterion for leading edge separation. *J. Fluid Mech.* **222**, 33-37.

An asymptotic approximation for a harmonic flow past a slender airfoil is constructed by matching the thin-airfoil solution and the solution for an apparent parabola in a uniform stream. The geometry of the flow near the leading edge, in particular the location of the stagnation point, depends on a parameter directly connected with the airfoil's angle of attack. Used as an input into Prandtl's boundary layer equations this data allows to predict critical angles of attack, at which boundary layer separation occurs, and thereby to look for the shapes delaying the leading edge separation.

THIN-AIRFOIL SUMMARY

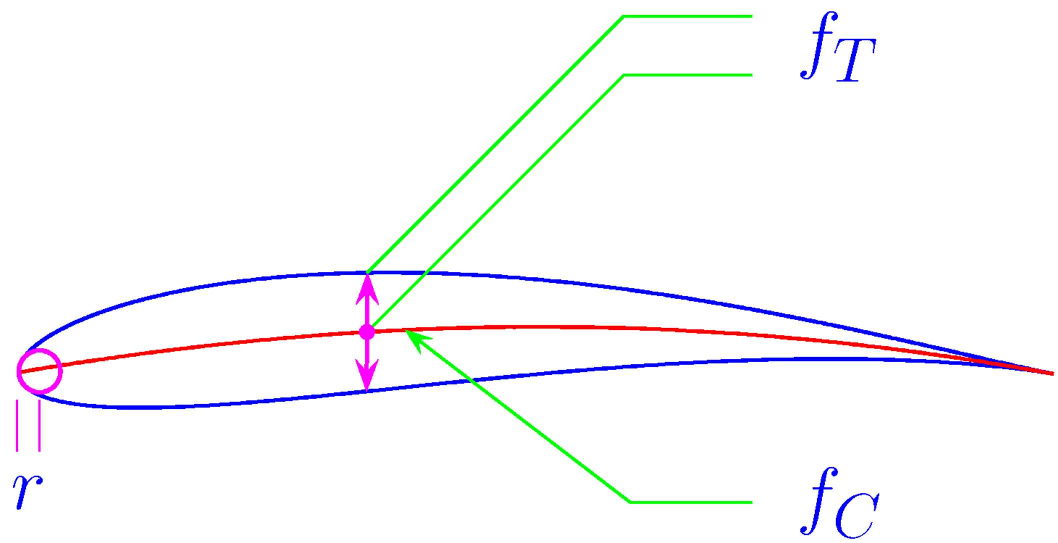
Consider a wing in a subsonic uniform stream:



An ideal flow shown on this picture meets the wing at angle α, called the *angle of attack*. At small angles α the viscous flow remains unseparated and the resulting lift is proportional to α. Hence a greater α generates a greater lift. However, we can increase α with the flow remaining unseparated only up to a certain value α_{critical}. Transition from the phase of unseparated flow to the the phase of separated flow is accompanied by a sudden loss of lift and is called *stall*. Delaying the stall to larger values of α is therefore one of the aims of a good aerodynamic design.

The outline of the wing's cross-section can be written as:

$$y = f_C(x) \pm f_T(x), \quad 0 \le x \le c,$$



where $f_C(x)$ is the middle line and
$f_T(x)$ describes thickness.

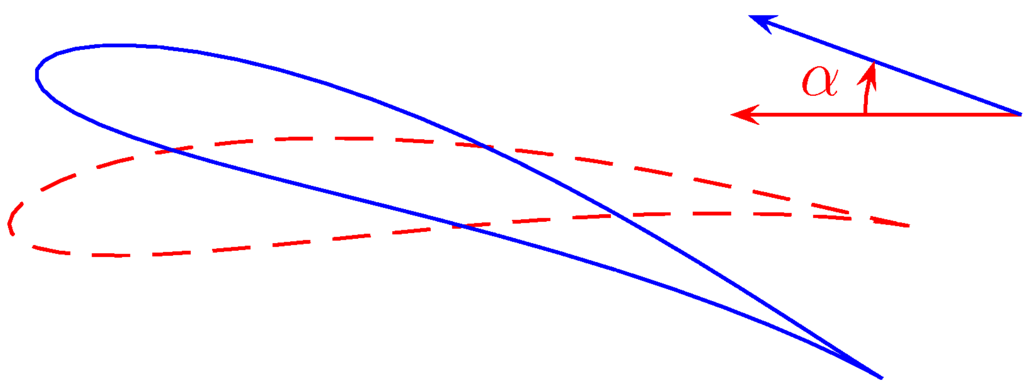
Denote by r the radius of the outline's curvature at the leading edge.

SMALL NOSE RADIUS OF CURVATURE

Assume that r is small and

$$
\begin{aligned}
f_T(x) &= (2r)^{1/2}(x^{1/2} + O(x^{3/2})) \\
\alpha &= O(\sqrt{r}) \\
f_C(x) &= \sqrt{r} F_C(x)
\end{aligned}
$$

where $F_C(x)$ depends on x only and is bounded together with its derivative. Keep the stream horizontal and rotate the airfoil by angle α.



Since r and α are small the outline of the turned airfoil can be written as

$$
\begin{aligned}
&y = -\alpha x + f_C(x) \pm f_T(x) \\
&0 \le x \le c.
\end{aligned}
$$

Consider the flow $\nabla(Ux+\phi)$ past this airfoil. The perturbation potential $\phi(x,y)$ satisfies the Laplace equation and the boundary conditions:

$$\phi_y = (U+\phi_x)y'(x) \text{ on } y = y(x), \qquad (1)$$

$$|\nabla\phi| \to 0, \text{ as } \sqrt{x^2+y^2} \to \infty, \qquad (2)$$

$$\text{Kutta condition} \qquad (3)$$

The Kutta condition is a requirement that the speed of the flow be finite at the trailing edge. It effectively decides on the value of circulation about the wing and therefore controls the lift.

If we use our assumptions about α, f_C and f_T, and expand the kinematic condition (1) near $y=0$, $x \in (0,c)$, retaining only $O(\sqrt{r})$ terms, we obtain the boundary condition:

$$\phi_y = U(-\alpha + f_C'(x) \pm f_T'(x)), \quad y = 0_\pm\,. \quad (4)$$

Combining conditions (2), (3) and (4) with $\nabla^2\phi = 0$ gives the "thin-airfoil" problem. The complex velocity potential

$$w(z) = \phi + i\psi$$

for this problem is known, its derivative (complex velocity) is

$$\frac{dw}{dz} = -\frac{U}{\pi i}\sqrt{\frac{z-c}{z}}\int_0^c \sqrt{\frac{s}{c-s}}\frac{(-\alpha + f_C'(s))}{(s-z)}ds - \frac{U}{\pi}\int_0^c \frac{f_T'(s)}{(s-z)}ds\,. \tag{5}$$

This solution fails in a neighbourhood of the leading edge giving infinite velocities. It must be replaced there with a properly matched perturbation term from the solution describing a flow past a parabola in a uniform stream.

SOLUTION NEAR LEADING EDGE

The flow near the leading edge can be approximately described by the complex velocity potential $[Z \equiv z/r]$:

$$F_{\text{inner}}(z) = rUf(Z)$$

where $f(Z) = Z + (\beta - i)(2Z - 1)^{1/2}$.

On the parabola $y^2 = 2rx$,

$$\operatorname{Im} F_{\text{inner}}(z) = rU\beta = \text{const},$$

hence $F_{\text{inner}}(z)$ is a complex potential for a uniform flow U past the parabola $y^2 = 2rx$.

The complex velocity for $F_{\text{inner}}(z)$ is

$$F'_{\text{inner}}(z) = U\left(1 + (\beta - i)(2Z - 1)^{-1/2}\right).$$

MATCHING

The $x \to 0_+$ behaviour of $w'(z)$ on airfoil's "surface" is given by

$$\frac{dw}{dz} = \pm U \left[(\alpha - \alpha_0) \sqrt{\frac{c}{x}} - i \sqrt{\frac{r}{2x}} \right] + \gamma(x, r), \qquad (6)$$

where $|\gamma(x,r)| \leq \text{const} \cdot \sqrt{r}$ and

$$\alpha_0 = \frac{1}{\pi} \int_0^c \frac{f_C'(s)}{[s(c-s)]^{1/2}} ds \qquad \left(\text{is } O(\sqrt{r})\right). \quad (7)$$

The complex velocity $F'_{\text{inner}}(z)$ on the surface $y = \pm\sqrt{2rx}$ can be written as

$$F'_{\text{inner}}(z) = U + U(\beta - i)/(i \pm \sqrt{2x/r}) \qquad (8)$$

Solution (8) is to approximate the flow in a small neighbourhood of the leading edge (distances $\sim r$), whereas (6) is to describe the perturbation potential at distances $\gg r$ from the leading edge.

Comparing (6) and the corresponding member from (8) in some intermediate region, e.g. taking $x \sim r^{1-\delta}$ ($0 < \delta < 1$), we find that the principal terms ($\sim r^{\delta/2}$) coincide if

$$\beta = (\alpha - \alpha_0) \left(\frac{2c}{r} \right)^{1/2} .$$

If we choose the initial orientation of the airfoil so as to generate f_C for which the integral (7) is zero, then this can be written as

$$\beta = \alpha \left(\frac{2c}{r} \right)^{1/2} . \qquad (9)$$

More details about this formula can be found in Ruban (1981) and Tuck (1991).

NON-PARABOLIC NOSES

Consider now the complex velocity of the form:

$$F'_{\text{inner}}(z) = U\left(1 + \frac{\beta - i}{\sqrt{2Z-1}} + \omega(Z)\right) \qquad (10)$$

where $\omega(Z)$ is analytic in the flow domain and is $O(Z^{-1})$ as $|Z| \to \infty$.

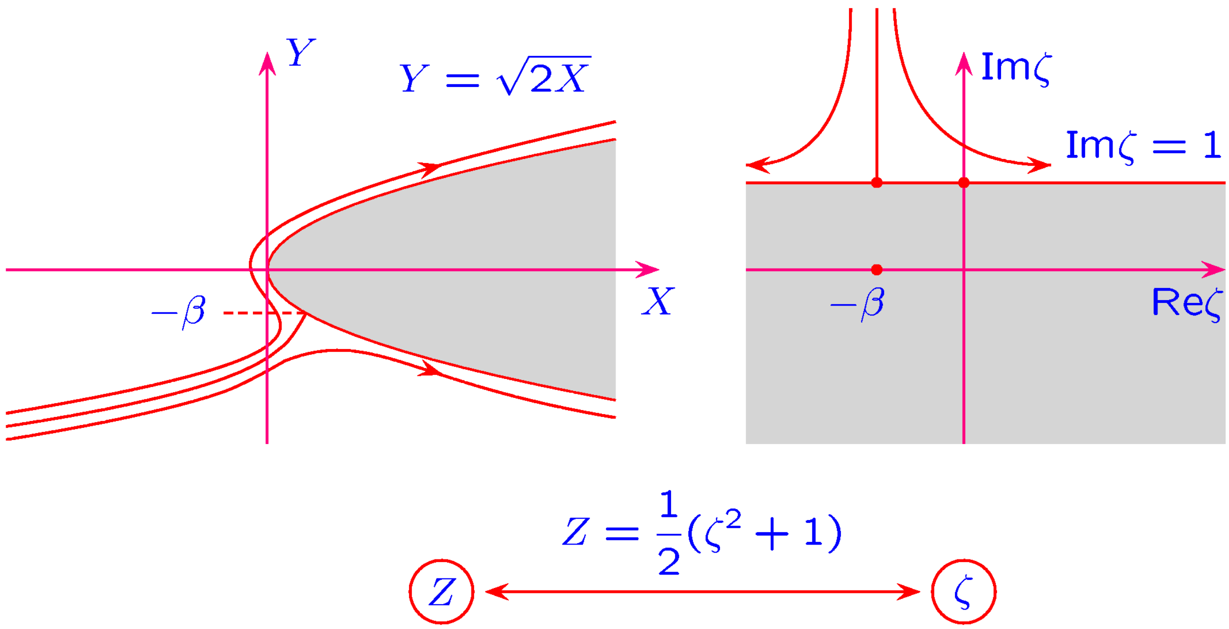
The matching procedure used in the case of a parabolic inner solution identically applies to the inner solution of this form since introduction of $\omega(Z)$ has no influence on the leading terms in the intermediate region. Thus the formula (9) relating parameter β and the angle of attack α can be used for (10) as well.

FAMILY OF NOSE SHAPES

The solution for a parabola can be derived by considering the complex velocity potential

$$f = \frac{1}{2}(\zeta + \beta - i)^2$$

and the conformal mapping $Z = \frac{1}{2}(\zeta^2 + 1)$.



This gives the complex velocity potential we used earlier:

$$f = \left(Z + (\beta - i)(2Z - 1)^{1/2}\right) .$$

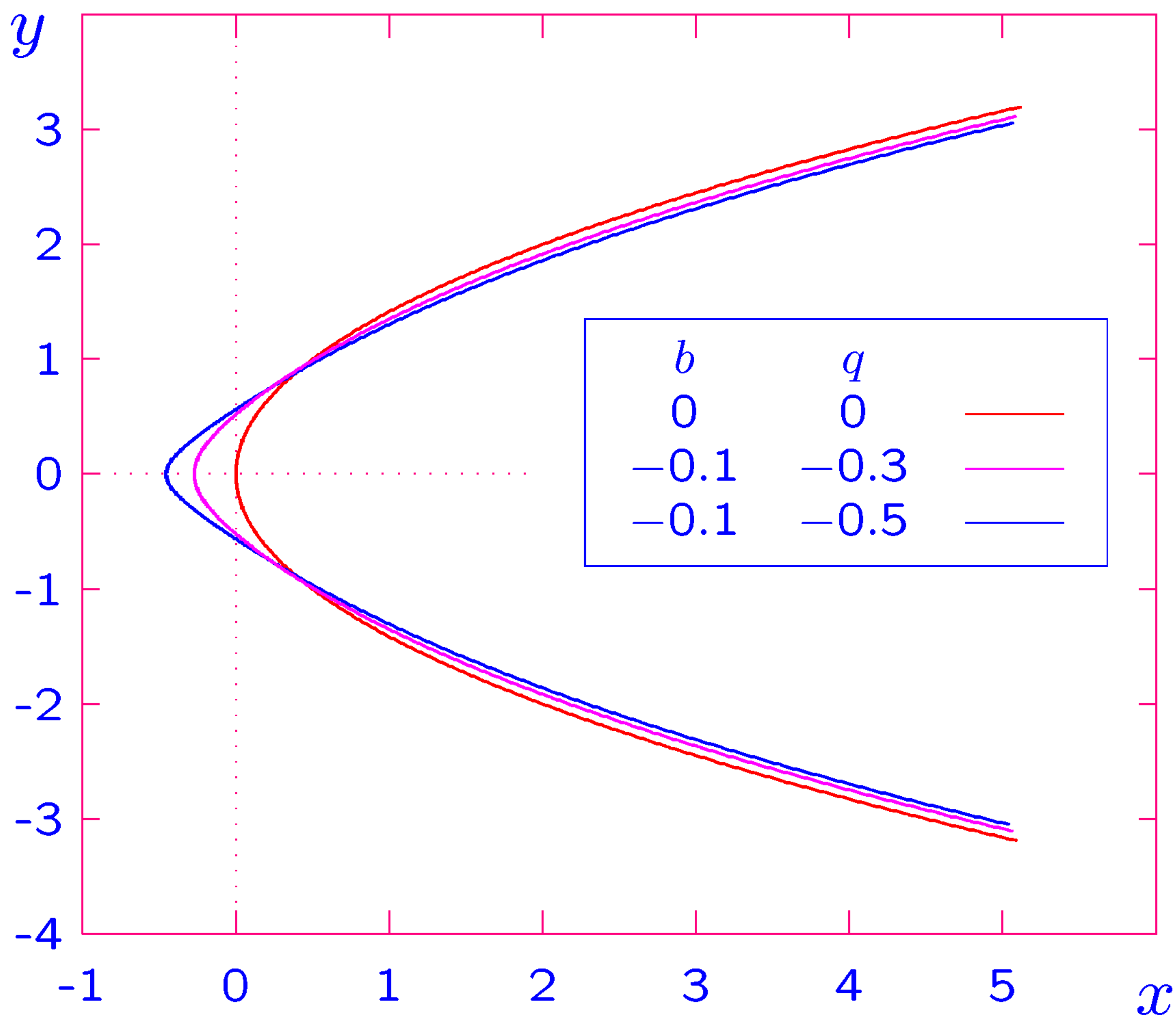
In order to obtain the flow satisfying (10), consider the complex velocity potential defined by the $f(\zeta)\,\&\,Z(\zeta)$ pair

$$f = \frac{1}{2}(\zeta + \beta - i)^2\,,$$
$$Z = \frac{1}{2}(\zeta^2 + 1) + \frac{p + iq}{[\zeta - (a + ib)]^m}$$

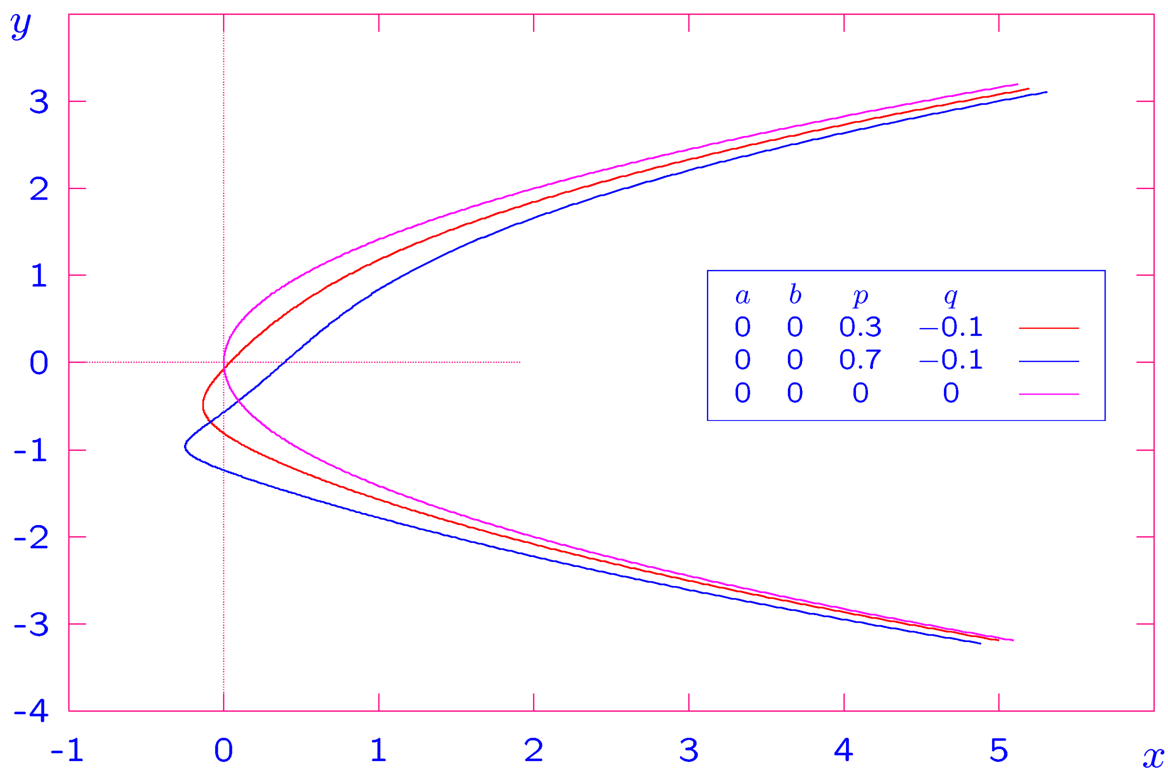
(m positive integer, $b < 1$).
This flow in the ζ plane is the same as before. In the physical plane, however, $f(Z)$ describes a flow past a shape distorted near the nose but asymptotically approaching parabola at large $|Z|$. For example, if $m = 1$, the parametric form of such a shape is

$$x = \frac{1}{2}t^2 + \frac{p(t-a) + q(1-b)}{(t-a)^2 + (1-b)^2}$$
$$y = t + \frac{q(t-a) - p(1-b)}{(t-a)^2 + (1-b)^2}$$

(setting $\zeta = t + i$, ($t \in (-\infty, \infty)$)).



Examples of symmetrical shapes
for $m = 1$.



Examples of non-symmetrical shapes
for $m = 1$.

Formally, assumptions of the adopted asymptotic approximation do not imply that the shape must become "parabolic" within a fixed bounded region. However, if we wish to ease requirements on smallness of r, we should consider the shapes which approach parabola as quickly as possible. In this sense $m = 1$ is not a particularly good choice as the resulting shape becomes parabolic at considerable distances from the nose. The curves corresponding to $m \geq 2$ quickly approach parabola, but, as we found out, do not give a significant delay in separation. We reached a compromise by considering the mapping

$$Z = \frac{1}{2}(\zeta^2 + 1) + \frac{1}{1 - ih\zeta} \cdot \frac{p + iq}{\zeta - (a + ib)}$$

This modification allows to achieve a better control as we increase β_0, with the shape changes remaining local to the nose when h is small.

BOUNDARY LAYER COMPUTATION

The non-dimensional equations governing steady 2D laminar flow of a viscous fluid near a curved surface are:

$$\frac{\partial u}{\partial s} + \frac{\partial v}{\partial n} = 0 \tag{11}$$

(continuity equation),

$$u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} = u_e\frac{du_e}{ds} + \frac{\partial^2 u}{\partial n^2} \tag{12}$$

(Prandtl's equation) where s and n are non-dimensional curvilinear coordinates measured along and normal to the surface starting from the stagnation point; u and v are respective velocity components.

The boundary conditions are

$$u(s,0) = v(s,0) = 0 \tag{13}$$

(no-slip and impermeability conditions) and

$$u(s,n) \to u_e(s) \text{ as } n \to \infty \tag{14}$$

(u matches the non-viscid velocity at the edge of the boundary layer).

Following Werle & Davis we introduce the Görtler variables

$$\xi = \int_0^s u_e ds \quad \text{and} \quad \eta = \frac{u_e}{\sqrt{2\xi}} n \, ,$$

in which (11) and (12) become

$$2\xi \frac{\partial F}{\partial \xi} + F + \frac{\partial V}{\partial \eta} = 0 \tag{15}$$

$$2\xi \frac{\partial F}{\partial \xi} + V \frac{\partial F}{\partial \eta} + P'(\xi)(F^2 - 1) = \frac{\partial^2 F}{\partial \eta^2} \tag{16}$$

where

$$u = u_e F \quad \text{and} \quad v = \frac{u_e}{\sqrt{2\xi}} V - \frac{\partial \eta}{\partial s} \sqrt{2\xi} F$$

and the pressure gradient input is

$$P'(\xi) = \frac{2\xi}{u_e} \frac{du_e}{d\xi} \, .$$

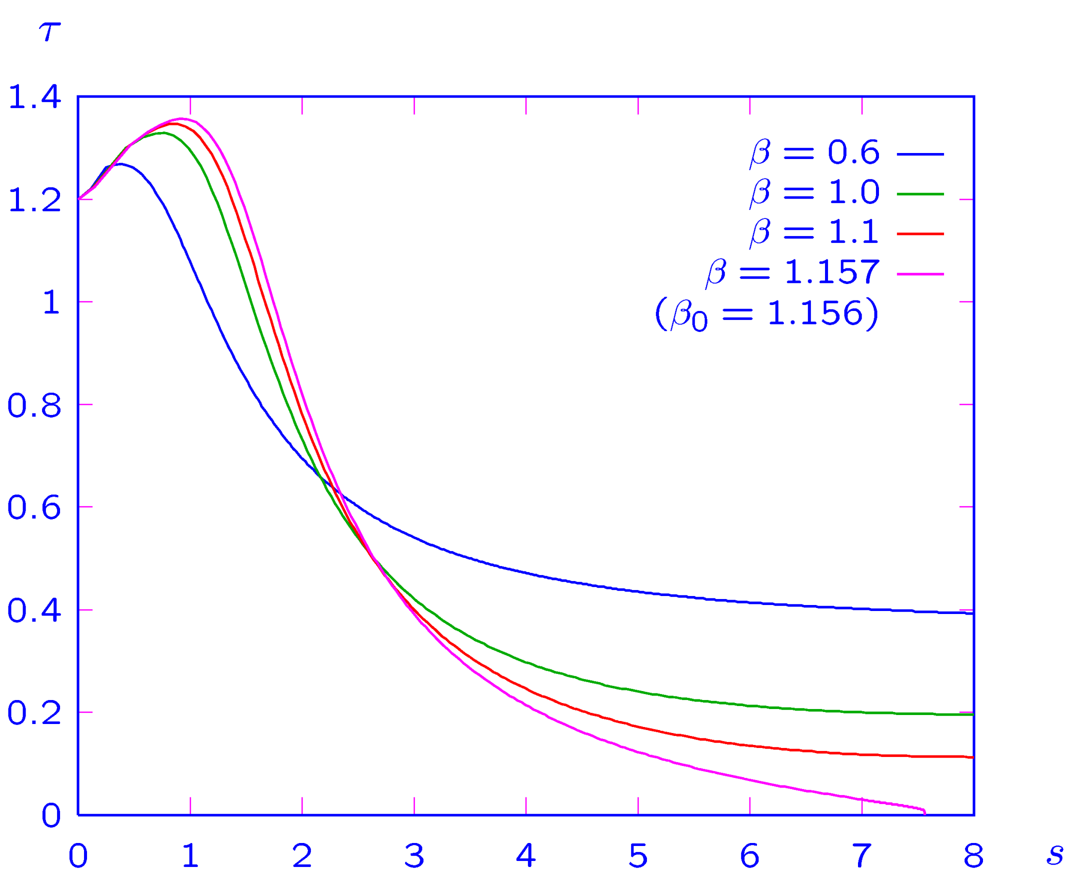
In these variables the boundary conditions take the form:

$$F(\xi, 0) = V(\xi, 0) = 0, \ \lim_{\eta \to \infty} F(\xi, \eta) = 1. \tag{17}$$

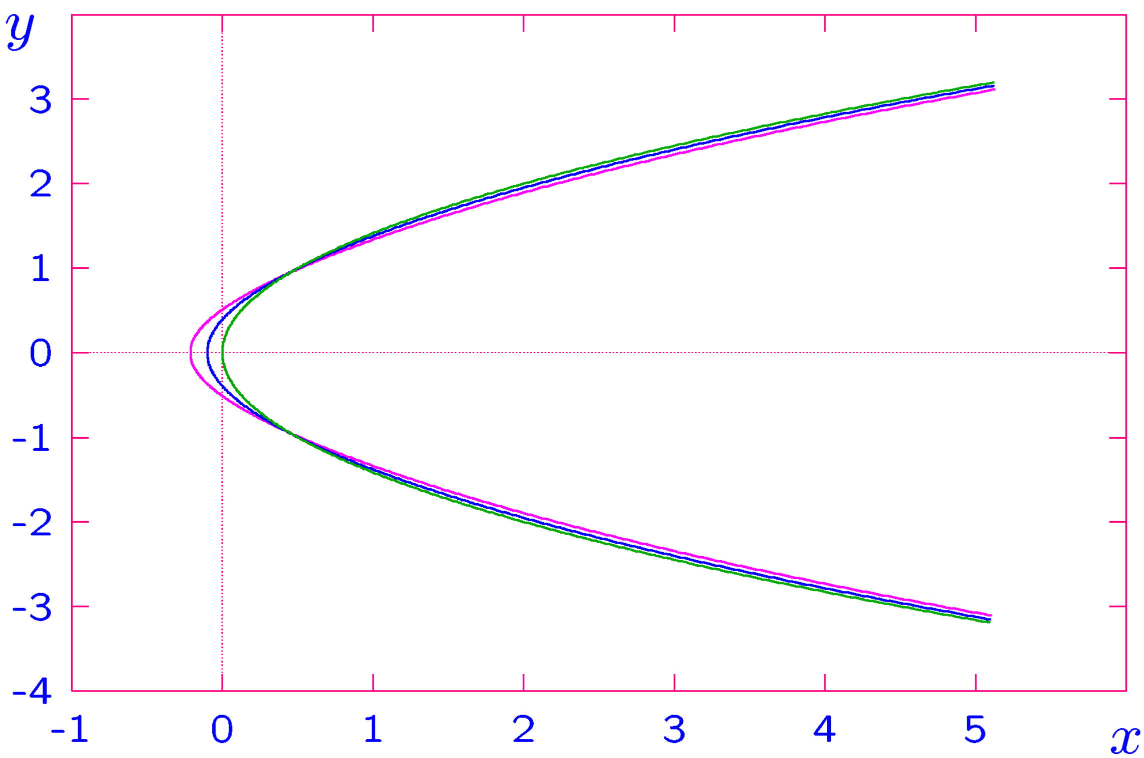
We solve the system of (15) and (16) subject to the boundary conditions (17) for the various shapes described above analyzing the results in terms of the skin friction factor

$$\tau = (\partial/\partial\eta)F|_{\eta=0} \ .$$

The flow remains unseparated if $\tau > 0$ for all ξ. The separation criterion is thus $\tau = 0$.

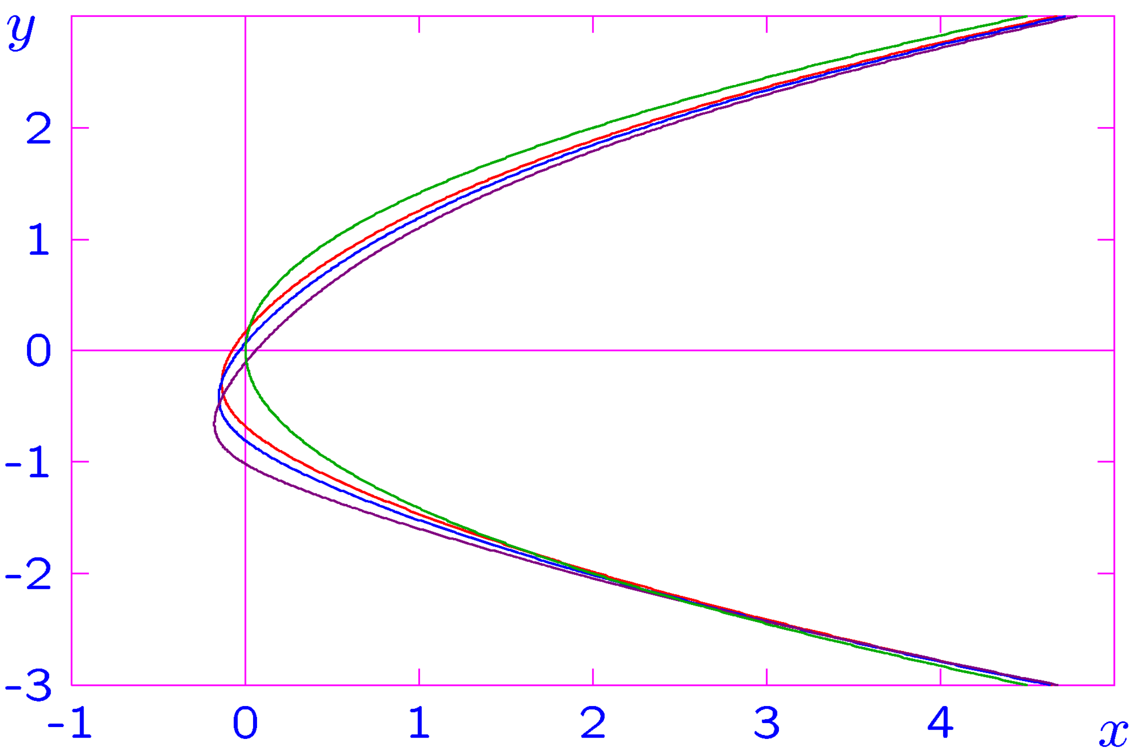


A sequence of symmetrical noses with increasing β_0



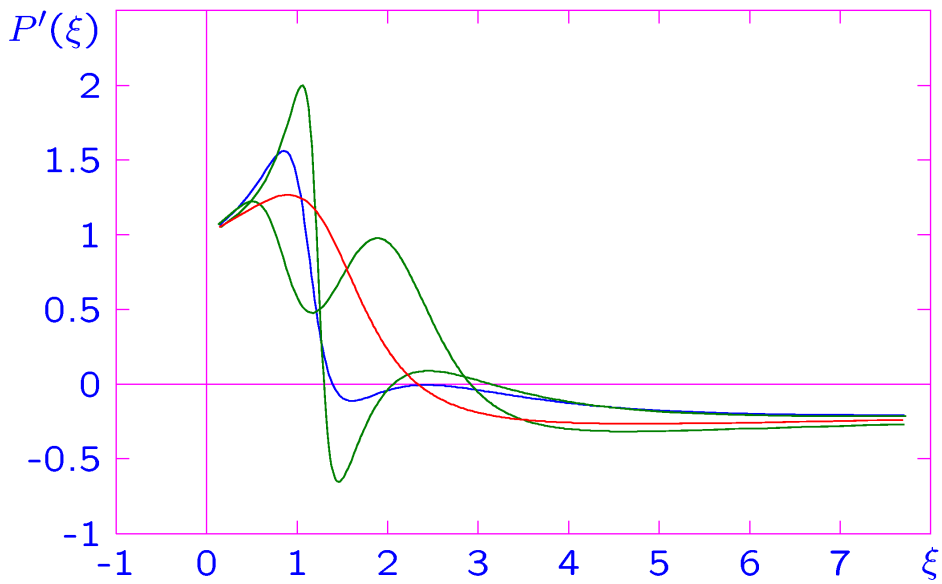
Line style	a	b	p	q	h	β_0
green line	0	0	0	0	0	1.156
blue line	0	-0.6	0	-0.2	0.25	1.210
magenta line	0	-0.8	0	-0.5	0.3	1.269

A sequence of non-symmetrical noses with increasing β_0.



Line style	a	b	p	q	h	β_0
green line	0	0	0	0	0	1.156
red line	-0.2	-0.4	0.27	-0.27	0.3	1.252
blue line	-0.2	-0.4	0.4	-0.29	0.25	1.270
purple line	-0.4	-0.4	0.6	-0.4	0.25	1.289

The empirical rule illustrated below facilitates the search for optimal shapes.



Taking $P'(\xi)$ for a parabola ($= P'_{\text{parab}}$ – shown with a red line) with maximum value of β ($= 1.156$) as a reference we consider a set of values a, b, p and q to be a "good" choice if (a) the plot $P'(\xi)$ for the same value of β lies above $P_{\text{parab}}(\xi)$ after the second intersection, and (b) the local minimum of $P'(\xi)$ is not lower than the local minimum of $P'_{\text{parab}}(\xi)$. Condition (b) is needed to avoid an early crisis, whereas condition (a) indicates delayed boundary-layer separation. The blue line in the diagram satisfies (a) and (b), whereas the green lines do not satisfy conditions (a) and (b).