SHIP-WAVE PATTERNS IN THE SPIRIT OF MICHELL

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Abstract J.H. Michell solved in 1898 the problem of a thin ship moving steadily forward in a calm sea, and was able, in spite of the total absence of computing equipment, to evaluate with 2-figure accuracy the triple integral for the resulting inviscid drag force or wave resistance. Michell's formulation allowed in principle determination of the actual ship-wave pattern, but this task was not completed for the whole wave field for about another century. Far-field waves can be computed with only marginally more difficulty than wave resistance, and some such computations appeared a few decades ago, but even then there are subtleties and fine details such as very short diverging waves that are difficult to capture with adequate accuracy. Waves near the ship are an order of magnitude harder to compute. We have produced very fast code for both near and far fields, that can determine a finely detailed pattern in about an hour on an inexpensive PC, a task that J.H. Michell would no doubt have carried out himself if he had such a device on his desk in 1898. A feature that Michell might not have included is an empirical dissipation factor for the far-field waves, incorporating an eddy viscosity coefficient which damps out some of the shortest diverging waves. Results are given for a destroyer hull where agreement with experiment is reasonable, and in particular is not necessarily worse than that for codes incorporating features such as nonlinearity and full viscous effects.

Keywords: ship waves, computation, eddy viscosity

THIN-SHIP THEORY

The free-surface elevation $z = Z(x, y)$ due to a thin ship with offsets $y = \pm Y(x, z)$ moving steadily at speed U in the $-x$ direction in water of infinite depth is

$$Z(x, y) = -\frac{2}{k_0} \iint_R Y_\xi(\xi, \zeta) G_x(x - \xi, y, z; \zeta) d\xi \, d\zeta \; . \qquad (1)$$

Here R is the centreplane $y = 0$ of the ship and $\phi = G(x, y, z; \zeta)$ is the velocity potential for a unit "Havelock" source at $(x, y, z) = (0, 0, \zeta)$,

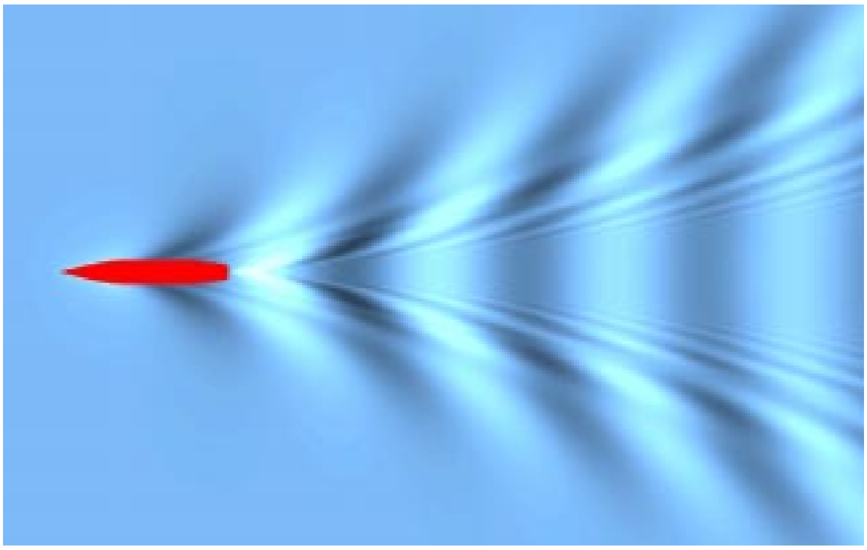

Figure 1. Computed wave pattern of a destroyer hull at 30 knots.

satisfying a linearised free-surface boundary condition $k_0\phi_z + \phi_{xx} = 0$ on $z = 0$, with $k_0 = g/U^2$, namely (Wehausen and Laitone, 1962, p. 484)

$$G(x,y,z;\zeta) = -\frac{1}{4\pi r} \qquad (2)$$
$$+ \quad \frac{1}{4\pi^2}\Re\int_{-\pi/2}^{\pi/2}\int_0^\infty e^{-ik(x\cos\theta+y\sin\theta)}\frac{k+k_0\sec^2\theta}{k-k_0\sec^2\theta}e^{k(z+\zeta)}dk\,d\theta$$

with r the distance to the source, and the k-integration path passing above the pole at $k = k_0\sec^2\theta$. The task of computing $Z(x,y)$ thus requires a total of four numerical integrations.

An alternative formula results from interchange of the order of (ξ,ζ) and (k,θ) integrations, namely

$$Z(x,y) = \frac{1}{\pi^2}\Re\int_{-\pi/2}^{\pi/2}\int_0^\infty e^{-ik(x\cos\theta+y\sin\theta)}\frac{k^2(P+iQ)}{k-k_0\sec^2\theta}\,dk\,d\theta \qquad (3)$$

where

$$P + iQ = -\frac{1}{ik\cos\theta}\int\!\!\int_R Y_\xi(\xi,\zeta)\;e^{ik\cos\theta\xi+k\zeta}\;d\xi\,d\zeta\;. \qquad (4)$$

This alternative is sometimes described (see e.g. Noblesse, 2000) as a “Fourier-Kochin” representation, although it is in the spirit of Michell’s (1898) wave-resistance formula.

Although there are still four integrations to perform, the alternative form has advantages when computing the elevation at a large number of points (x, y), since the P, Q functions can be computed and stored once and for all. Nevertheless, direct quadruple numerical integration remains prohibitively expensive for all but benchmark checking purposes.

FAR FIELD AND WAVE RESISTANCE

When x is large and positive, i.e. far behind the ship, the dominant contribution to the wave elevation comes from the residue at the pole $k = k_0 \sec^2\theta$ in (3), namely

$$Z_F(x,y) = -\frac{2k_0^2}{\pi}\,\Re\, i\int_{-\pi/2}^{\pi/2} e^{-ik_0\sec^2\theta(x\cos\theta+y\sin\theta)}\sec^4\theta(P+iQ)\,d\theta\,. \tag{5}$$

Equation (5) shows that the far-field wave pattern Z_F is the sum over all possible directions θ, of plane waves of amplitude proportional to $P + iQ$, or energy proportional to $P^2 + Q^2$. The total energy yields the wave resistance

$$R = \frac{2\rho g^4}{\pi U^6}\int_{-\pi/2}^{\pi/2}(P^2+Q^2)\sec^5\theta\,d\theta\,. \tag{6}$$

In combination with (4), this is Michell's (1898) triple integral for the wave resistance of a ship. The similarity between (5) and (6) suggests that the task of computation of the far-field wave elevation at each separate point (x, y) will be comparable to that of computing one value for the wave resistance, a daunting prospect for a field of 100,000 points.

We have developed an efficient routine (Tuck 1987, Tuck and Lazauskas 1999), for evaluation of Michell's integral (6),(4), which uses Filon's quadrature (Abramowitz and Stegun, 1964, p. 890) in the ξ-direction, a Filon-like method in the ζ-direction, and Simpson's rule for the θ-integral. The ξ-wise Filon quadrature captures the correct decay as $|\theta| \to \pi/2$, when the integrand of (4) with $k = k_0 \sec^2\theta$ becomes a rapidly oscillating function of ξ, whereas conventional (e.g. Simpson) quadratures fail to produce the correct decay of the diverging part of the wave spectrum. This program computes the wave resistance of a typical ship to 4-figure accuracy in less than a fifth of a second on an inexpensive 500MHz PC.

Essentially the same numerical methods that have been successful for wave-resistance computations are now used to compute the far-field wave elevation $Z_F(x, y)$. In addition, a special algorithm as in Tuck *et al* (1971) also captures the stationary-phase character of this integral as $x, y \to \infty$, thus allowing uniform accuracy of computation as we move far away from the ship. The PC time to compute a single far-field wave elevation is then about a quarter of a second. However, because the P, Q functions can be stored and used repetitively, the time to compute a wave field of 100,000 points is only about the same as for about 4000 separate single-point calculations, i.e. about 15 minutes.

NEAR FIELD

Suppose we now write $G = G_F + G_L$ where G_F is the "far-field" portion of the Havelock source potential (2), defined for all x as the residue component when $x > 0$ and zero otherwise, and G_L is the remaining "local" potential which decays rapidly as we move away from the source in all directions. Then correspondingly the wave elevation can for all (x, y) values be separated into $Z = Z_F + Z_L$. For x values aft of the stern of the ship, $Z_F(x, y)$ is as described in the previous section, and for x values ahead of the bow it vanishes. Between bow and stern, a similar computational procedure can be adopted for $Z_F(x, y)$, albeit with a slight loss of efficiency, because the P, Q functions need to be re-computed for each new x (but not y) value.

It remains to compute the local field $Z_L(x, y)$, and in principle this remains a formidable quadruple-integration task. Fortunately, a significant part of this task has already been done for us, since Newman (1987) has provided economised polynomial approximations for G_L. Hence we need merely substitute this polynomial code into the original formula (1) and carry out the (ξ, ζ) integrations by Simpson's rule (no Filon treatment is needed as the local integrand is not rapidly varying). Nevertheless, one cannot entirely escape the fact that near-field computations require more arithmetic effort than far-field, and the local part of the computation always dominates computer times, typically by a factor of about four. The net effect is that the total PC time to compute a complete 100,000-point field is about an hour.

RESULTS AND EDDY VISCOSITY

Figure 1 is a sample of a typical computed wave field, for a destroyer travelling at 30 knots. This Figure was produced on a 400 by 250 grid, i.e. 100,000 separate wave elevations were computed in the region shown. A nice feature of the present program is that we can now "zoom in" on arbitrarily-fine details of this pattern, although what we actually do is repeat the computations with the 100,000 points distributed over a smaller region. In contrast, the resolution of "panel" methods is set in advance by the choice of the number of panels. Figure 2 shows a four times magnified view of the centre of Figure 1. Only the top (starboard) half is immediately relevant to this discussion.

Of particular interest is the diverging-wave structure, which manifests itself as very short long-crested "ripples", which are especially apparent as we move away from the large crests near the Kelvin angle, toward the track of the ship. This fine structure is genuine in inviscid theory and cannot be captured without careful attention to accuracy and numerical convergence — in particular it tends to be lost without the Filon treatment of the ξ−integral in (4).

However, in one sense it should be lost! In the real world, such very short waves are damped out by viscosity. It is possible in an empirical manner to account for this effect as follows.

Suppose we insert a multiplicative factor in the θ-integrand for the far-field waves (5), of the form

$$\exp\left[-2\nu k_0^2 \frac{x}{U} \sec^4\theta\right]. \tag{7}$$

This factor is taken from Lamb (1932, p. 625), for damping of plane waves. The actual kinematic molecular viscosity of water $\nu \approx 10^{-6}\mathrm{m}^2\mathrm{s}^{-1}$ is far too small to see any effect. However, oceanographically relevant (e.g. Cushman-Roisin, 1994, pp. 45, 63, 72 and Mei, 1983, pp. 9, 413) eddy viscosities of the order of $\nu = 0.005\ \mathrm{m}^2\mathrm{s}^{-1}$ do damp out the shortest diverging waves as $|\theta| \to \pi/2$.

Figure 2 shows in its bottom (portside) half, the effect of including the factor (7). Our investigations have shown there is little dependence of the free-surface elevation on the actual value of the viscosity, unless ν is varied over several orders of magnitude. We have used the exaggerated value $\nu = 0.05\ \mathrm{m}^2\mathrm{s}^{-1}$ in Figure 2 to enhance the comparison. Compared with the zero-viscosity top half of the figure, the finest diverging ripples in the pattern have disappeared. It is these ripples, which propagate almost perpendicular to the ship's track, that the eddy viscosity affects most, due to the $\sec^4\theta$ term in (7). The effect is similar to a low-pass filter used by Sclavounos *et al* (1993). Larger viscosities remove even more detail, eventually even inappropriately damping transverse waves, and $\nu = 0.005\ \mathrm{m}^2\mathrm{s}^{-1}$ seems a good compromise. An eddy viscosity of this magnitude has essentially no other effect on the wave pattern.

COMPARISONS

The particular destroyer hull ("DTMB Model 5415") of Figures 1–2 is one for which extensive experimental towing-tank tests were done in 1991, and comparisons made with various then state-of-the-art computer programs, as reported by Lindenmuth *et al* (1991). Raven (1996)

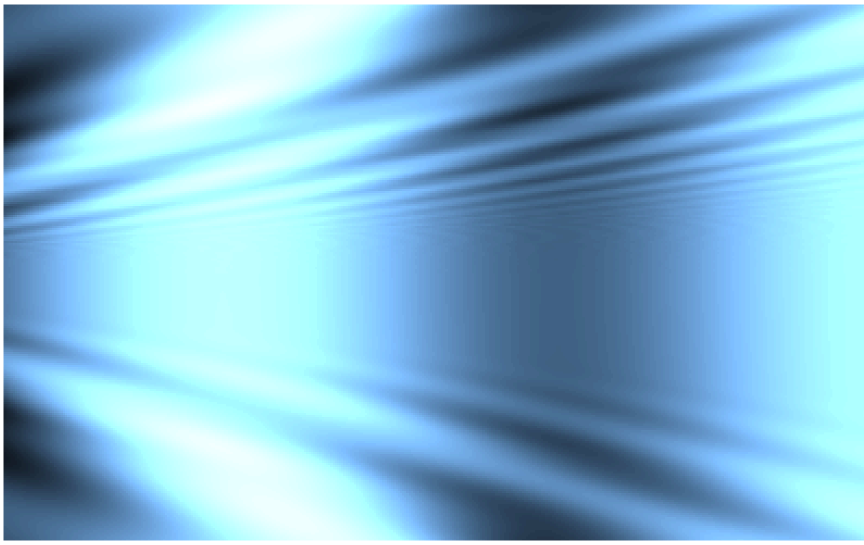

Figure 2. Waves made by a destroyer – doubly expanded view. Top half (starboard side of ship) is at zero viscosity, bottom half (port side) is with $\nu = 0.05\ \mathrm{m^2 s^{-1}}$.

has since also provided computations for this ship, and Noblesse (2000) has recently reviewed extended developments of one of the most successful of the 1991 codes. Figure 3 shows these results together with our computations, for a wave trace along a parallel cut at a fixed value of y.

No attempt has been made to distinguish the (lightly-sketched) curves for the various competing codes in this Figure; the "clutter" of these curves is in itself a useful measure of the extent to which computational tools can predict ship waves quantitatively. All codes do quite a reasonable job considering the magnitude of the task, and ours is at least as good as any of the others, sometimes significantly better. Some of the other codes are nonlinear and some include viscous effects throughout the flow field, but there is no evidence that discrepancies with this particular experiment (which could be due to deficiencies in the experimental measurements rather than in the computations) are reduced significantly by including such effects.

MULTIHULLS

The present program can also be used for multihulls (Tuck and Lazauskas, 1998). Figure 4 shows an example of an asymmetric catamaran where there is almost total wave cancellation on the starboard side.

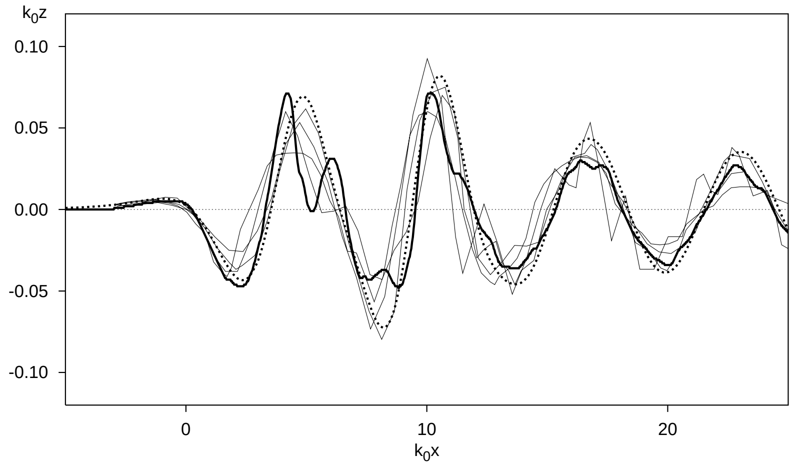

Figure 3. Comparison between experiment (solid curve), the present code (dashed curve) and other codes (light curves) for a destroyer hull.

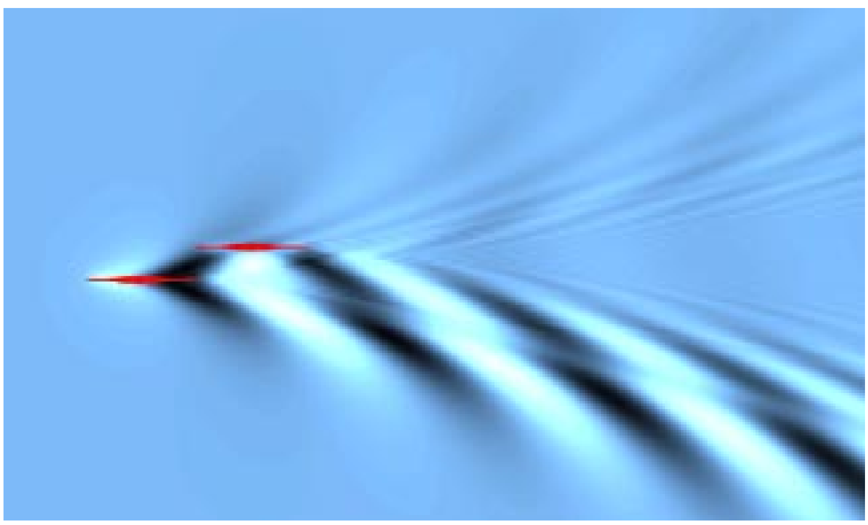

Figure 4. An asymmetric “Weinblum” catamaran.

Acknowledgments

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