ADELAIDE
UNIVERSITY
AUSTRALIA

Analysis on Manifolds

Venue: Ingkarni Wardli building

Date: September 31 - October 04

Organisers: Mathai Varghese, Diarmuid Crowley,
Matthias Ludewig, Guanheng Chen

Titles and Abstracts

Monday September 30

Venue: Room 7.15, Ingkarni Wardli building

Time: 10:00-10:55

Speaker: Weiping Zhang

Title: Positive scalar curvature on manifolds and foliations

Abstract: we will discuss various generalizations of the Lichnerowicz vanishing theorem to the case of foliations, including a recent joint result with Guangxiang Su on noncompact foliations.

Time: 11:00-11:55

Speaker: Peter Hochs

Title: A localised equivariant index for proper actions and an APS index theorem.

Abstract: Roe defined a localised version of the coarse index of an elliptic operator that is invertible outside a subset Z of the manifold M it is defined on. An equivariant version of this index was defined for proper and free actions by discrete groups by Xie and Yu. With Guo and Mathai, we extended this to proper actions by any locally compact group G. If Z/G is compact, then this index takes values in the K-theory of the group C^* algebra of G, and generalises the Baum-Connes analytic assembly map. It also generalises an equivariant index of Callias-type operators constructed earlier by Guo. Another special case is an equivariant index for proper, cocompact actions on manifolds with boundary, generalising the Atiyah-Patodi-Singer (APS) index and its equivariant version. With Bai-Ling Wang and Hang Wang, we proved an equivariant APS index theorem in this context. This applies to actions by, for example, finitely generated discrete groups and semisimple Lie groups.

Time: 13:30-14:25

Speaker: Diarmuid Crowley

Title: The Topological Period-Index Conjecture for 8-manifolds

Abstract: The (topological) Brauer group of a space X is the torsion subgroup of the the third integral cohomology of X. The index of a Brauer class can be defined via an index map in the twisted K-theory of X.

The topological period-index problem of Antieau and Williams is the problem of relating the index of a Brauer class to its order (which is called the period) to the dimension of X.

In this talk I will explain the motivation of this problem from algebraic geometry (when X is the analytic space underlying a smooth complex variety) and also recent progress on the period-index problem when X is an 8-manifold. This is joint work with Christian Haesemeyer and Xing Gu.

Time: 14:30-15:25

Speaker: Romina M. Arroyo

Title: The long-time behaviour of the pluriclosed flow on Lie groups

Abstract: The *pluriclosed flow* is a geometric flow that evolves pluriclosed Hermitian structures (i.e. Hermitian structures for which its 2-fundamental form satisfies $\partial\bar{\partial}\omega = 0$) in a given complex manifold. The aim of this talk is to discuss the asymptotic behaviour of the pluriclosed flow in the case of left-invariant structures on Lie groups. More precisely, invariant structures on 2-step nilmanifolds and almost abelian solvmanifolds. We will analyze the flow and explain how a suitable normalization converges to *pluriclosed solitons*, which are self-similar solutions to the flow. Moreover, we will show that some of those limits are shrinking solitons, which is an unexpected feature in the solvable case. We will also exhibit the first example of a homogeneous manifold on which a geometric flow has some solutions with finite extinction time and some that exist for all positive times.

This is a joint work with Ramiro Lafuente (The University of Queensland).

Time: 16:00-16:55

Speaker: Saskia Roos

Title: Dirac operator under collapsing to a smooth manifold

Abstract: We consider a sequences of spin manifolds with bounded curvature and diameter that collapses to a smooth manifold. In that setting we show that a part of the spectrum of the Dirac operator converges to an explicitly given twisted Dirac operator with a symmetric potential. Moreover, we give an example of collapsing sequences where the spectrum of the Dirac operator converges to the spectrum of the Dirac operator on the limit space.

Tuesday October 01

Venue: Room 7.15, Ingkarni Wardli building

Time: 10:00-10:55

Speaker: Nigel Higson

Title: A rescaled spinor bundle on the tangent groupoid

Abstract: I shall present an account of Ezra Getzler's approach to the Atiyah-Singer index theorem that starts from Alain Connes' tangent groupoid. The ingredients are a rescaled spinor bundle on the tangent groupoid of a spin manifold M that is defined in more or less the same way that the tangent groupoid is defined from M itself, and a convolution structure on the smooth, compactly supported sections of the rescaled spinor bundle that extends Connes' concept of the convolution algebra of scalar functions on a smooth groupoid. The convolution algebra of sections supports a smooth family of supertraces, the existence of which is a sort of "index theorem without a Dirac operator." This is joint work with Zelin Yi (Chern Institute of Mathematics).

Time: 11:00-11:55

Speaker: Hang Wang

Title: An equivariant Atiyah-Patodi-Singer index theorem for proper actions

Abstract: Consider a Dirac type operator on a manifold having product metric near its boundary, acted by a locally compact group properly cocompactly and isometrically. Assume that the boundary Dirac operator has isolated spectrum at 0. We show that the Dirac operator admits an equivariant index in the K-theory of the group C*-algebra. By taking traces we obtain an L^2-APS index formula and delocalized APS index formulas in the setting of proper actions. Relationships with secondary invariants and positive scalar curvature metrics will be reviewed. This is joint work with Peter Hochs and Bai-Ling Wang.

Student Talks

Time: 13:30-14:00

Speaker: David Brook

Title: Higher twisted K-theory

Abstract: Higher twisted K-theory is a recent generalisation of topological K-theory stimulated by results of Ulrich Pennig and Marius Dadarlat, who use an operator algebraic approach to provide geometric representatives for all abstract twists of K-theory. It involves a larger class of twists than Rosenberg's twisted K-theory - which has ties to string theory - and as such it is expected to be of physical interest in the realm of M-theory. In this talk, I will present an introduction to higher twisted K-theory and explain why this new construction is expected to yield meaningful results. In particular, I will outline computational techniques including the Mayer-Vietoris sequence and a twisted Atiyah-Hirzebruch spectral sequence, and present constructions of explicit geometric representatives for twists in special cases.

Time: 14:00-14:30

Speaker: Jaklyn Crilly

Title: T-duality, Courant algebroids and the exotic.

Abstract: In this talk we introduce the notion of an exotic Courant algebroid and detail how the T-duality mapping between Courant algebroids extends to a duality mapping between the exotic Courant algebroids. This extension is then compatible with the T-duality mapping between exotic differential forms. Joint work with V. Mathai.

Time: 14:30-15:00

Speaker: Johnny Lim

Title: Analytic Pontryagin Duality

Abstract: Let X be a smooth compact manifold. The Universal Coefficient Theorem in K-theory with coefficients in $\mathbb{R}/\mathbb{Z}$ asserts that there is an isomorphism between the $\mathbb{R}/\mathbb{Z}$ K-theory group $K^i(X, \mathbb{R}/\mathbb{Z})$ and the Hom group $\mathrm{Hom}(K_i(X), \mathbb{R}/\mathbb{Z})$. We study an explicit analytic duality pairing in

the even case which implements the isomorphism. We propose a model of the group $K^0(X, \mathbb{R}/\mathbb{Z})$. Together with the even Baum-Douglas geometric K-homology $K_0(X)$, we formulate an analytic pairing comprises of the Dai-Zhang eta-invariant of a certain Dirac-type operator and a topological term. This analytic pairing is well-defined and non-degenerate, thus giving a robust $\mathbb{R}/\mathbb{Z}$ invariant. As a motivation, we study two special cases of the analytic pairing in cohomology $H^1(X, \mathbb{R}/\mathbb{Z})$ and $H^2(X, \mathbb{R}/\mathbb{Z})$.

Time: 15:00-15:30

Speaker: Lachlan McDonald

Title: Dynamical invariants of foliated manifolds

Abstract: A foliation of a manifold M specifies the set of integral submanifolds (referred to in foliation theory as "leaves") to a system of differential equations on M. The "holonomy" of integral curves in a foliated manifold gives rise to an intrinsic dynamical system, whose topology has implications for the global behaviour of solutions to the corresponding system of PDEs. In this talk, I will give a primer on foliations and their associated dynamical invariants, as well as discuss how this information can be accessed via path integrals and noncommutative index theory.

Time: 16:00-16:30

Speaker: Nicholas McLean

Title: Title: Index Theory on Non-Compact Manifolds

Abstract: For compact manifolds, it is well known that all elliptic operators are Fredholm if they act between the correct function spaces. On non-compact manifolds, the ellipticity of an operator is no longer sufficient to ensure the operator is Fredholm and thus an index, in this case, becomes harder to define. In this talk, I am to highlight the subtleties which occur on non-compact manifolds when trying to do similar constructions, and then give a brief overview of a method developed by Roe in his paper, titled "An index theorem on open manifolds", to circumvent the issues that arise due to non-compactness. Finally, I will talk about my own research which involves combining Roe's approach with the equivariant version of the index

theorem to produce an index that is more general than the one presented in Roe's paper.

Time: 16:30-17:00

Speaker: Csaba Nagy

Title: The smooth classification of complete intersections

Abstract: Complete intersections are important examples of smooth complex projective varieties. The smooth classification of complete intersections is a long standing problem in differential topology, which is organised by a statement known as the Sullivan Conjecture.

I this talk I will report on a proof of the Sullivan Conjecture in complex dimension 4 and discuss relationships between index theory and the Sullivan Conjecture for spin complete intersections in complex dimension 5. This is part of joint work with Diarmuid Crowley.

Time: 17:00-17:30

Speaker: Ahnaf Tahabub

Title: Bardeen-Cooper-Schreiffer Theory on Hyperbolic Surfaces

Abstract: According to usual BCS theory, superconductivity is a consequence of electrons forming bosonic Cooper pairs due to the presence of the cationic lattice inside the metal. There is an energy gap which prevents the Cooper pairs from breaking into free electrons when the temperature is low enough. I will generalise this mechanism to a lattice on the hyperbolic half plane by constructing the BCS functional and the corresponding gap equation. Then I'll state a theorem relating this hyperbolic BCS functional to the hyperbolic GL functional. Finally, I'll state some peculiar properties that arise from the GL equations on hyperbolic space. This includes the decay rate of external magnetic fields inside a hyperbolic superconductor and the geometrical shift in critical temperature.

Wednesday October 02

Venue: Room B18, Ingkarni Wardli building

Time: 10:00 -10:55

Speaker: George Willis

Title: Analysis on Trees and Buildings

Abstract: The idea that regular trees are discrete generalisations of the hyperbolic plane has motivated much work on discrete geometry and the representation theory of disconnected groups. A paper by A. Figà-Talamanca and M. Picardello concerning harmonic analysis on free groups via their actions on regular trees was my first introduction to this idea.

Trees are the 'rank 1' case of discrete geometries known as *buildings*, which are discrete generalisations of higher dimensional symmetric spaces introduced by Jacques Tits. Harmonic analysis on free groups and trees has been extended to harmonic analysis on automorphism groups of buildings in papers by Cartwright, Młotkowski, Parkinson, Steger and others.

Automorphism groups of trees and buildings are totally disconnected and locally compact. The structure of the groups is encoded in the geometry of the trees and buildings, and their harmonic analysis is consequently closely related to that geometry as well.

This talk will survey aspects of this circle of ideas from the point of view of the shift of the focus of my research from harmonic analysis to the structure of groups.

Time: 11:00 -11:55

Speaker: Fei Han

Title: On $string^c$ structures and modular invariants

Abstract: String structure is a higher version of spin structure, which also corresponds to spin structure on loop space. In this talk, we will introduce in topological ways of $string^c$ structure, the higher version of $spin^c$ structure, their definitions, loop space meaning and counting of $string^c$ structures. Under $string^c$ conditions, we construct modular invariants, prove Liu's type vanishing theorem and show some applications. This is based on

our joint work with Haibao Duan and Ruizhi Huang.

Time: 13:30 -14:25

Speaker: Alan Carey

Title: KO valued Spectral Flow (joint work with Chris Bourne, Matthias Lesch and Adam Rennie)

Abstract: This is an old idea going back to a paper of Lott in 1988. Somewhat surprisingly nothing has been done with this notion since then. One possible reason is that the definition is not a workable one. Prompted by applications to topological insulators we revisited the concept. We have a comprehensive analytic treatment that I will discuss. It explains some anomalous examples that have arisen in applications.

Time: 14:30 -15:25

Speaker: Adam Rennie

Title: Curvature in noncommutative geometry

Abstract: Over the last decade Connes and coworkers have developed an approach to curvature for the noncommutative torus based on the heat kernel expansion. I will present two alternative approaches, one a specialisation of the other, which provide more complete information about curvature, and are more generally applicable. Examples include the Podles sphere and theta deformations of classical examples. This is joint work with Walter van Suijlekom and Bram Mesland.

Time: 16:00 -16:55

Speaker: Galina Levitina

Title: Pre-spectral triples: noncommutative geometry for symmetric non-self-adjoint operators

Abstract: We introduce the notion of a pre-spectral triple (A, H, D), which is a generalisation of a spectral triple where $D$is no longer required to be

self-adjoint, but only symmetric. Despite having weaker assumptions, pre-spectral triples provide an abstract setting for noncompact noncommutative geometry with boundary. Joint work with A. Connes, E. McDonald, F. Sukochev and D. Zanin.

Thursday October 03

Venue: Room 7.15, Ingkarni Wardli building

Time: 10:00 -10:55

Speaker: Malabika Pramanik

Title: Restriction of eigenfunctions to sparse sets on manifolds

Abstract: Abstract: Given a compact Riemannian manifold (M, g) without boundary, we consider the restriction of Laplace-Beltrami eigenfunctions to certain subsets Γ of the manifold. How do the Lebesgue L^p norms of these restricted eigenfunctions grow? Burq, Gerard, Szvetkov and independently Hu studied this question when Γ is a submanifold.

In joint work with Suresh Eswarathasan, we extend earlier work to the setting where Γ is an arbitrary fractal set, and differential geometric methods no longer apply. We obtain sharp growth estimates for the restricted eigenfunctions that rely on measure theoretic properties of Γ. Our results are sharp for large p, and are realized for large families of sets Γ that are random and Cantor-like.

Time: 11:00 -11:55

Speaker: Leo Tzou

Title: Rigidity of Asymptotically Conic Spaces

Abstract:

We look at how the behaviour of geodesics dictate the geometry of asymptotically conic spaces. We study the properties of the X-ray transform for asymptotically Euclidean or conic Riemannian metrics and show its injectiivity. We also discuss how conjugate points play a role in determining the structure of such geometries. Joint with with Guillarmou, Mazzucchelli, Lassas.

Friday October 04

Venue: Room 7.15, Ingkarni Wardli building

Time: 10:00 -10:55

Speaker: Maxim Zabzine

Title: Generalized Kähler potential

Abstract: I will review the basic facts about generalized Kähler geometry and its relation to Kähler geometry. I will explain the problem of generalized Kähler potential and present the solution based on the notion of Morita equivalence for Poisson structures.

Time: 11:00 -11:55

Speaker: Guo Chuan Thiang

Title: Edge-following topological states

Abstract: Recently, physicists have been able to create "topological states" localised on the boundary of a 2D system, which have rather crazy properties: they fill up spectral gaps in the boundary-less 2D problem, and have quantised propagation along the boundary regardless of its shape. I will explain how the index theory of semigroup C^*-algebras explains this phenomenon.