A quantitative Gidas-Ni-Nirenberg-type result for the p-Laplacian via integral identities

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Abstract

The study of geometric properties of solutions of Partial Differential Equations is of great interest. One of the most celebrated results in this topic is the one obtained by B. Gidas, W. M. Ni and L. Nirenberg in [2], where the authors establish radial symmetry to nonnegative solutions of the problem

$$\begin{cases} -\Delta u = f(u) & \text{on } B, \\ u = 0 & \text{on } \partial B, \end{cases}$$

where $B \subset \mathbb{R}^N$ (with $N \geqslant 2$) is a ball. Later on, several authors went on to generalize this result in several directions.

In this talk, I will give an overview of the history of the problem of proving the radial symmetry of solutions to Partial Differential Equations. I will also present a quantitative version of a Gidas-Ni-Nirenberg-type symmetry result involving the p-Laplacian. Quantitative stability is achieved here via integral identities based on the proof of rigidity established by J. Serra in 2013, which extended to higher dimensions and the p-Laplacian operator an argument proposed by P. L. Lions in dimension 2 for the classical Laplacian.

In passing, we obtain a quantitative estimate for the measure of the singular set and an explicit uniform gradient bound.

This work was done in collaboration with S. Dipierro, G. Poggesi, and E. Valdinoci in [1] and will appear in the Journal of Functional Analysis.

References

[1] S. Dipierro, J. G. da Silva, G. Poggesi, and E. Valdinoci, *A quantitative Gidas-Ni-Nirenberg-type result for the p-Laplacian via integral identities*, arXiv preprint arXiv:2408.03522;
To appear in the Journal of Functional Analysis. ↑1

[2] B. Gidas, W. M. Ni, and L. Nirenberg, *Symmetry and related properties via the maximum principle*, Comm. Math. Phys. **68** (1979), no. 3, 209–243. MR544879 ↑1