Moduli Spaces, Representation Theory, and Quantization

Titles and Abstracts

Andrea Appel

Title: Differential twists, universal algebras and quantization.

Abstract: It is well-known that the Drinfeld-Jimbo quantum groups are examples of the Etingof-Kazhdan universal quantization of Lie bialgebras applied to the case of symmetrisable Kac-Moody algebras. In this talk, I will recall the basic of Etingof-Kazhdan construction and, in particular, its "universality", that is, its realization in the terms of PROPs and universal algebras. I will then show how this approach can be adapted to the case of Kac-Moody algebras, providing a universal setting for the realization of the dynamical KZ equation, a "differential" twist, and, ultimately, a "differential" realization of the quantum group. This is based on past and ongoing works with V. Toledano Laredo.

Sjoerd Beentjes

Title: The crepant resolution conjecture for Donaldson-Thomas invariants.

Abstract: Donaldson-Thomas (DT) invariants are integers that enumerate curves in a given Calabi-Yau 3-fold. Let X be a 3-dimensional Calabi-Yau orbifold, and let Y be a crepant resolution of its coarse moduli space. When X satisfies the Hard Lefschetz condition, i.e., when the fibres of the resolution are at most 1-dimensional, the Crepant Resolution Conjecture (CRC) of Bryan-Cadman-Young gives a precise relation between the generating functions of DT invariants of X and Y. I will discuss joint work with John Calabrese and Jørgen Rennemo in which we interpret the relation of the CRC as an equality of rational functions, and prove it using the motivic Hall algebra and Joyce's wall-crossing formula.

Ben Davison

Title: Preprojective cohomological Hall algebras.

Abstract: I will explain various features of the preprojective CoHA, a kind of universal algebra of correspondences generalising the algebras of endomorphisms of cohomology of quiver varieties considered by Nakajima. In particular I will focus on features of this algebra that become visible after viewing it as a dimensional reduction of the (Kontsevich-Soibelman)

critical CoHA associated to a related 3-dimensional Calabi-Yau category. Many nice features emerge from this view, e.g. a cocommutative coproduct, an embedding in a related shuffle algebra, a formula for the graded dimension of the algebra, a flat non cocommutative deformation, a geometric doubling procedure, the PBW theorem due to joint work with Sven Meinhardt, and an isomorphism with the Yangian considered by Maulik and Okounkov.

Iordan Ganev

Title: Quantum multiplicative quiver varieties at a root of unity.

Abstract: A common phenomenon when quantizing at roots of unity is the appearance of sheaves of Azumaya algebras on moduli spaces. We describe a mechanism for constructing such sheaves via a version of quantum Hamiltonian reduction that relies on Lusztig's quantum Frobenius homomorphism and the notion of Poisson orders. This procedure allows us to describe, and often compute explicitly, various Azumaya loci. We illustrate the general method through a particular instance, namely, the case of quantized multiplicative quiver varieties, which are moduli spaces of quiver representations constructed via Hamiltonian reduction along a group-valued moment map.

Sam Gunningham

Title: Character stacks, skein modules, and q-geometric representation theory.

Abstract: I will discuss applications of geometric representation theory to topological and quantum invariants of character stacks. In particular, I will explain how generalized Springer correspondence for class D-modules and Koszul duality for Hecke categories encode surprising structure underlying the homology of character stacks of surfaces (joint work with David Ben-Zvi and David Nadler). I will then report on some work in progress with David Jordan and Pavel Safronov concerning a q-analogue of these ideas. The applications include an approach towards a conjecture on the finite dimensionality of skein modules and methods for computation.

Kazuki Hiroe

Title: Confluence of singular points of Fuchsian equations and deforma-

tion of star-shaped quiver varieties.

Abstract: Moduli spaces of Fuchsian differential equations on the Riemann sphere can be realized as star-shaped quiver varieties due to Crawley-Boevey. On the other hand, confluence of regular singular points creates irregular singular points. In this talk, we shall see that the confluent process induces deformation of moduli spaces of Fuchsian equations on the Riemann sphere and that of star-shaped quiver varieties simultaneously via Crawley-Boevey's correspondence. We note that every fiber of our deformation is still a quiver variety with another quiver. Looking at the deformation of Heun's differential equation for example, we can construct a holomorphic family of quiver varieties with Euclidean quivers of type D_4 (star-shaped), A_4, A_3, A_2 as fibers.

Kohei Iwaki

Title: Painlevé τ-function and topological recursion

Abstract: Topological recursion was originally formulated as an algorithm to compute the large N expansion of correlation/partition functions of matrix models from their spectral curves. We will apply the topological recursion to a family of genus one spectral curves, and show that the discrete Fourier transform of the topological recursion partition function gives the tau-function of the Painlevé equation. We will also discuss the associated direct monodromy problem through the exact WKB method.

Qiongling Li

Title: Higgs bundles in Hitchin section and certain Hitchin fibers.

Abstract: In the moduli space of $SL(n, C)$-Higgs bundles over a closed Riemann surface of genus at least 2, we have two n types of half-dimensional sublocus: the Hitchin section and the Hitchin fiber. We study harmonic maps corresponding to Higgs bundles in the Hitchin section and in the Hitchin fiber at $(q_2, 0, \ldots, 0)$. The Hitchin section intersects each Hitchin fiber at $(q_2, 0, \ldots, 0)$ at exactly a Fuchsian point. In this talk, we show that in terms of the energy density of harmonic maps, the Fuchsian point provides the minimum in the Hitchin section and provides the maximum in its own Hitchin fiber. Part of this work is joint with Song Dai.

Anton Mellit

Title: The curious hard Lefschetz property for character varieties.

Abstract: I will talk about a way to decompose a character variety of a Riemann surface of arbitrary rank with prescribed semisimple generic local monodromies into cells which look like a product of an affine space and a symplectic torus. This can be thought of as abelianization. As an application, we deduce the curious hard Lefschetz property conjectured by Hausel, Letellier and Rodriguez-Villegas, which claims that the operator of cup product with the class of the holomorphic symplectic form is an isomorphism between complementary degrees of the associated graded with respect to the weight filtration on the cohomology.

Luca Migliorini

Title: Supports of the Hitchin fibration on the reduced locus.

Abstract: I'll discuss some work in progress in collaboration with M.A. de Cataldo and J. Heinloth.

Let C be a nonsingular projective curve of genus > 1, and let n and d be two coprime integers. Given the moduli space $\mathscr{M}$ of stable Higgs bundles (E, Φ), where E is a vector bundle on C of rank n and degree d, and $\Phi\colon E \to E \otimes K_C$ is the Higgs field, there is an associated Hitchin fibration $\chi : \mathscr{M} \to \mathscr{A}$, where $\mathscr{A}$ is the affine space parameterizing the spectral curves $C_a \subset T^\vee C$ associated to Higgs bundles. The map χ is projective, hence, by the decomposition theorem of Beilinson Bernstein Deligne and Gabber, $R\chi_*\mathbb{Q}_{\mathscr{M}}$ splits in a direct sum of (shifted) intersection cohomology complexes. We determine completely the supports of those intersection cohomology complexes which intersect the locus $\mathscr{A}_{red}$ parameterizing *reduced* spectral curves. We show in particular that, for every partition $\lambda = (\lambda_1, \cdots, \lambda_r)$ of n, there is a string of summands appearing in the decomposition of $R\chi_*\mathbb{Q}_{\mathscr{M}}$ which is supported on the locus $\mathscr{A}_\lambda$ where the spectral curve splits into r reducible components mapping to C with degrees $\lambda_1, \cdots, \lambda_r$. The proof relies on the technique of "higher discriminants", developed in collaboration with V. Shende, and on some results on the universal family of compactified jacobians, due to V. Shende, F. Viviani and myself.

Paul Norbury

Title: Deformation of curves inside a Poisson surface.

Abstract: Beginning with the deformation space B of a curve inside a Poisson surface, Kontsevich and Soibelman construct a quadratic lagrangian submanifold in an associated infinite dimensional affine symplectic space. The quadratic lagrangian submanifold is equivalent to a so-called Airy structure which gives the initial data for topological recursion. I will use these ideas to prove a relationship between topological recursion and the variation of the period matrix of the curves inside the family B, generalising a result of Baraglia and Huang.

Tony Pantev

Title: Moduli of flat connections on smooth varieties (I and II)

Abstract: This is a report on recent joint works with Dima Arinkin and Bertrand Toën. We study the derived geometry of the moduli of local systems and flat bundles on a smooth but not necessarily proper complex algebraic variety.

In the Betti case we show that these moduli carry natural shifted Poisson structures, generalizing the well known case of curves. I will explain constructions of symplectic leaves for these Poisson structures arising from fixing local monodromy, or formal monodromy and Stokes data and at infinity. I will discuss a new feature, called strictness, which appears as soon as the divisor at infinity has non-trivial crossings.

In the de Rham case we introduce the notion of a formal boundary of a smooth variety, and define a restriction to the boundary map between derived moduli of flat bundles. I will discuss representability results for the geometric fibers of this map and will explain why the restriction to the boundary morphism comes equipped with a canonical shifted Lagrangian structure.

Szilárd Szabó

Title: P = W and Simpson's conjecture for Painlevé spaces.

Abstract: We use explicit understanding of the geometry of moduli spaces of irregular singular Higgs bundles and of wild character varieties corresponding to the Painlevé systems to compare the perverse polynomial of the cohomology on the Dolbeault side to the weight polynomial on the Betti side (numerical version of P = W). In the P6 case, we use abelianization to prove Simpson's conjecture about the ends of these moduli spaces.

Kenji Ueno

Title: Abelian conformal field theory and $SL(2,\mathbb{C})$ conformal blocks on curves of genus 2

Abstract: I will discuss conformal field theory with gauge symmetry $SL(2,\mathbb{C})$ on curves of genus 2 and associated connections. Relationship with abelian conformal field theory will be discussed.

Dimitri Wyss

Title: Non-Archimedean integrals on the Hitchin Fibration

Abstract: Non-archimedean or p-adic integration is an analytic tool to study rational points of algebraic varieties over finite fields. Dener-Loser and Batyrev have realized that this can be used in some cases to study the topology of complex algebraic varieties. We apply this idea to the moduli spaces of G-Higgs bundles $M(G)$ and show in particular, that for a pair of Langlands dual groups the corresponding moduli spaces have the same non-archimedean volume.

As a geometric application we find an agreement of (stringy) Hodge numbers of $M(SL_n)$ and $M(PGL_n)$ as predicted by a conjecture of Hausel-Thaddeus. For general G this leads to a new proof of the geometric stabilization theorem, a key ingredient in Ngô's proof of the fundamental lemma. This is joint work with Michael Groechenig and Paul Ziegler.