CONFORMAL AND SYMPLECTIC GEOMETRY

COMMON THREADS AND GLOBAL ASPECTS

TITLES AND ABSTRACTS

Anton Alekseev
University of Geneva
Monday 11:00 – 12:00

Title. *Moduli of flat connections, Kashiwara-Vergne and Batalin-Vilkovisky*

Abstract. Let S be an oriented surface of genus g and G a connected Lie group with Lie algebra $Lie(G)$ equipped with an invariant scalar product. The moduli of flat connections $\mathcal{M}(S,G)$ carries the Atiyah-Bott symplectic form. For $G = GL(n)$, this symplectic form can be encoded by the Goldman bracket on the space $F(S)$ spanned by homotopy classes of free loops on S. The Goldman bracket is defined in terms of intersections of curves on S. Turaev defined a cobracket on $F(S)$ using self-intersections of curves. We speculate that Turaev cobracket is related to a Batalin-Vilkovisky structure on $\mathcal{M}(S,G)$ in the case when G is Lie supergroup and the scalar product on $Lie(G)$ is odd. This speculation is supported by the study of the Kashiwara-Vergne problem.

The talk is based on a joint work in progress with F. Naef, J. Pulmann and P. Severa.

David Baraglia
University of Adelaide
Wednesday 14:30 – 15:20

Title. *Special Kähler geometry and topological recursion*

Abstract. The base of a complex algebraic integrable system carries a natural Kähler metric and a natural affine structure which together constitute what is known as special Kähler geometry. In this talk we will focus on the case of the Hitchin integrable system. We show that the special Kähler geometry may be computed using the theory of Eynard-Orantin topological recursion. In particular, we consider the Donagi-Markman cubic, which measures the difference between the Levi-Civita connection and the affine connection, and show that it is given by an Eynard-Orantin invariant. This talk is based on joint work with Zhenxi Huang.

Henrique Bursztyn
IMPA
Wednesday 9:30 – 10:30

Title. *Dirac structures: origins and recent applications*

Abstract. Just as symplectic and Poisson structures naturally arise in Hamiltonian mechanics, Dirac structures were introduced around 1990 by T. Courant and A. Weinstein to provide a geometric framework for mechanical systems with constraints. Dirac structures provide a unified viewpoint to several geometrical structures, and a key role in the theory is played by the so-called Courant brackets. Despite its original motivation in geometric mechanics, recent developments in "Dirac geometry" are related to a broad range of topics in mathematics and mathematical physics, including Lie theory, generalized complex geometry, group-valued moment maps, etc. The talk will give an introduction to Dirac structures, including their main examples and recent applications.

Andreas Cap
University of Vienna
Monday 9:30 – 10:30

Title. *Parabolic contact structures with a view towards symplectic geometry*

Abstract. Starting from the example of conformal structures, I will outline some general features of parabolic geometries, focusing on the class of parabolic contact structures. Apart from the fact that these structures exhibit most of the features of general parabolic geometries, they have a direct connection to a class of geometric structures which have an underlying (conformally) symplectic structure and to special symplectic connections, that I will outline in the last part of the talk.

Boris Doubrov
Belarusian State University
Thursday 9:30 – 10:30

Title. *Homogeneous Levi non-degenerate hypersurfaces in* $\mathbb{C}^3$

Abstract. We classify all (locally) homogeneous Levi non-degenerate real hypersurfaces in $\mathbb{C}^3$ with non-trivial infinitesimal stabilizer. The main idea is to use the complexification of such surfaces, which turns them into complete systems of PDEs of 2nd order, also having a transitive symmetry algebra with a stabilizer. The classification itself is based on the algebraic model and allows to distinguish the tubular models and those, which correspond to homogeneous hypersurfaces in $\mathbb{A}^3$.

Jesse Gell-Redman
University of Melbourne
Thursday 14:30 – 15:20

Title. *Index of Dirac-type operators on pseudomanifolds*

Abstract. We construct and study the index bundle of families of Dirac-type operators on pseudomanifolds with iterated wedge metrics; these are manifolds which undergo an iterated conic degeneration, both geometric and topological, and they include products of cone edge spaces. This involves an extension of the b-calculus of Melrose and the edge calculus of Mazzeo to this iterated setting. We go on to derive a formula for the Chern character of the index bundle, extending work of Bismut.

Joint with Pierre Albin at UIUC

Peter Hochs
University of Adelaide
Thursday 15:30 – 16:20

Title. *Spin^c quantisation*

Abstract. The geometric quantisation of a group action on a symplectic manifold was defined by Bott as the equivariant index of a Dirac operator on the manifold. The quantisation commutes with reduction principle is an idea from physics that yields a geometric expression for the decomposition of such an index, or quantisation, into irreducible representations of the group acting. Various versions of this principle were proved after Guillemin and Sternberg's first result for compact Kähler manifolds in 1982. In a breakthrough paper from 2014, Paradan and Vergne showed that this principle is not just true for symplectic manifolds, but also for the much more general Spin^c manifolds. In that generality the link with physics is no longer there, so it is remarkable that the principle still holds. This talk is an overview of this development, and includes joint work with Mathai and with Song.

Claude LeBrun
Stony Brook University
Wednesday 11:00 – 12:00

Title. *Anti-Self-Dual 4-Manifolds, Quasi-Fuchsian Groups, and Almost-Kähler Geometry*

Abstract. If a smooth manifold M admits a symplectic form, it also admits Riemannian metrics g that are related to the symplectic form by means of an adapted almost-complex structure. Such metrics are said to be almost-Kähler, because they are Kähler if and only if the almost complex structure is integrable. If M is compact and 4-dimensional, one can then show that the conformal classes of almost-Kähler metrics sweep out an open subset in the space of the conformal classes. This provides a natural tool for exploring difficult global problems in 4-dimensional conformal geometry, leading to non-trivial results and motivating broader conjectures in the subject.

However, this technique certainly has its limitations. For example, if a 4-manifold admits scalar-flat Kähler metrics, these can be deformed into anti-self-dual almost-Kähler metrics, and these then sweep out an open set in the moduli space of anti-self-dual conformal structures. One might somehow hope that this subset would also turn out to be closed, and so sweep out entire connected components in the moduli space. Alas, however, this simply isn't true! In this talk, I'll explain recent joint work with Chris Bishop that constructs a large hierarchy of counter-examples by studying the limit sets of quasi-Fuchsian groups.

Thomas Leistner
University of Adelaide
Friday 11:00 – 12:00

Title. *The ambient obstruction tensor and conformal holonomy*

Abstract. The obstruction tensor is a conformally covariant tensor that obstructs the existence of an analytic Ricci flat ambient metric in the sense of Fefferman and Graham. In the talk, I will describe a new relation between the obstruction tensor and the holonomy of the normal conformal Cartan connection. This relation implies several results on the vanishing and the rank of the obstruction tensor, for example for conformal structures with twistor spinors. As the main tool we introduce the notion of a conformal holonomy distribution whose integrability is closely related to the exceptional conformal structures in dimensions five and six that were found by Nurowski and Bryant. This is joint work with Andree Lischewski from the Humboldt-University Berlin.

Rui Loja Fernandes
University of Illinois at Urbana-Champaign
Tuesday 9:30 – 10:30

Title. *Associativity and Integrability*

Abstract. A fundamental result of Lie theory is Lie's Third Theorem: every finite dimensional Lie algebra integrates to a Lie group. This result fails for infinite dimensional Lie algebras (e.g., Banach Lie algebras) and it also fails for (finite dimensional) Lie algebroids, but every reasonable Lie algebra (finite or infinite dimensional) integrates to a local Lie group and every Lie algebroid integrates to a local Lie groupoid. On the other hand, a classical theorem of Mal'cev states that a local group is enlargeable to a group if and only if it is global associative. This talk will be an introduction to Lie algebroids and groupoids, focusing on the failure of Lie's Third Theorem and its relationship to the failure of associativity.

Heather Macbeth
Massachusetts Institute of Technology
Monday 15:30 – 16:20

Title. *A gluing construction for Kähler-Ricci solitons*

Abstract. I will explain the construction of a large new family of steady Ricci solitons. The solitons are Kähler, and the underlying Calabi-Yau manifolds are equivariant crepant resolutions of $\mathbb{C}^n/G$, where G is a finite subgroup of $SU(n)$. This is joint work with Olivier Biquard.

Vladimir Matveev
Friedriech-Schiller-Universität Jena
Friday 9:30 – 10:30

Title. *Symplectic geometry of Finsler metrics of constant curvature*

Abstract. I will discuss Finsler metrics of positive constant flag curvature (definition, previous results and geometry will be recalled) on closed 2-dimensional surfaces. The main result is that the geodesic flow of such a metric is symplectically conjugate to that of a Katok metric. Recall that Katok metrics are easy and well-understood examples of two-dimensional Finsler metrics of positive constant flag curvature; I explain what they are describe their geodesic flows. As a corollary, either all geodesics are closed, and at most two of them have length less than the generic one, or all geodesics but two are not closed; in the latter case there exists a Killing vector field.

The proof goes through finite dimensional integrable systems, the main technical step is that the geodesic flow of such a Finsler metric is integrable (in fact, in all dimensions). I then use results on sympectic geometry of integrable systems (e.g. action-angle variables) and of Zoll metrics.

The results are almost contained in the papers arXiv:1710.03736 , arXiv:1710.01281 coauthored in different constellations with R. Bryant, P. Foulon, S. Ivanov and W. Ziller.

Thomas Mettler
Goethe-Universität Frankfurt
Wednesday 13:30 – 14:20

Title. *Convex projective structures, holomorphic curves and hyperbolic metrics*

Abstract. Not every projective structure arises from the Levi-Civita connection of a Riemannian metric. Locally, every 2D projective structure does however arise from a conformal (or Weyl) connection. Surprisingly, the relevant PDE turns out to be equivalent to finding a holomorphic curve into a suitable complex surface. One might wonder about examples of 2D projective structures which do not globally arise from a conformal connection. It turns out that if a properly convex projective surface arises from a conformal connection, then it arises from the Levi-Civita connection of a hyperbolic metric. This is in parts joint work with Gabriel Paternain.

Michael Murray
University of Adelaide
Monday 13:30 – 14:20

Title. *JNR monopoles and holomorphic spheres*

Abstract. I will explain what a JNR monopole is and how the JNR data gives a simple formula for the holomorphic sphere of a hyperbolic monopole defined in a paper of mine with Paul Norbury and Michael Singer. This is joint work with Paul Norbury.

Katharina Neusser
Charles University
Monday 14:30 – 15:20

Title. *C-projective equivalence in Kähler geometry*

Abstract. While a projective structure on a manifold is given by a class of affine connections that have the same (unparametrised) geodesics, a c-projective structure on a complex manifold is given by a class of affine complex connections that have the same "J-planar" curves. In this talk we will be mainly concerned with c-projective structures induced by Kähler metrics (via their Levi-Civita connections) and present some work on the geometric and topological consequences of the existence of at least two c-projectively equivalent Kähler metrics. An application of these considerations is a proof of the Yano–Obata conjecture for complete Kähler manifolds—a metric c-projective analogue of the conformal Lichnerowicz conjecture. This talk is based on joint work with Calderbank–Eastwood–Matveev, and with Matveev.

Yuri Nikolayevsky
La Trobe University
Thursday 13:30 – 14:20

Title. *Conformal (and "genuine") Osserman Conjecture and conformal relatives of symmetric spaces*

Abstract. The spectrum of the Jacobi operator of a flat or a rank-one symmetric space is constant on the unit tangent bundle. In the late 80's, Robert Osserman conjectured that the converse is also true. At present, the Osserman Conjecture is resolved in the affirmative in all dimensions, with just one exception in which it is still open. A similar conjecture, with the curvature tensor replaced by the Weyl tensor, is known as the Conformal Osserman Conjecture. In the first part of the talk, I will give an overview of the current state of knowledge about both the Osserman Conjecture and its conformal counterpart. In the second part, we will discuss the following question lying in the overlap of the theory of curvature-homogeneous spaces and the classical Weyl-Schouten Theorem: if a Riemannian manifold has "the same" Weyl tensor as a given homogeneous space, is it necessarily conformally equivalent to it?

Andrew Swann
Aarhus University
Thursday 11:00 – 12:00

Title. *Toric geometry of G_2-manifolds*

Abstract. In symplectic geometry the Delzant construction gives a full classification of compact symplectic $2n$-manifolds with a Hamiltonian action of an n-torus. These ideas extend well to complete manifolds with a hyperKähler structure also providing a full classification and including examples with infinite topology. It is therefore natural to seek similar construcions for other Ricci-flat geometries. For 7-manifolds with holonomy contained in G_2, I will describe how the idea of multi-moment map may be used to start to build a structure theory for complete examples with appropriate torus symmetry. This is joint work with Thomas Bruun Madsen.

Gang Tian
Princeton University
Tuesday 11:00 – 12:00

Title. *Long-time behavior of Kähler-Ricci flow*

Abstract. This talk concerns my recent work joint with Zhenlei Zhang. I will start with an introduction on Kähler-Ricci flow. Then I will discuss our solution for a ten-years conjecture on long-time behavior on Kähler-Ricci flow. Our solution replies on a relative volume comparison theorem we developed for Ricci flow. This volume comparison generalizes the Bishop-Gromov volume comparison and Perelman's non-collapsing theorem.

Hang Wang
University of Adelaide
Wednesday 15:30 – 16:20

Title. *Conformal Invariant and Equivariant Index Theorem*

Abstract. We compute a family of conformal invariants associated with conformal diffeomorphisms and equivariant characteristic classes using local equivariant index formula in conformal geometry. The construction of the invariants involves various tools from noncommutative geometry. This is joint work with Raphael Ponge.