Please note: Question 6 (marked with an asterisk) should be handed in as part of **Assignment 3**, *due by* **noon on Friday 29 October (end Week 12)**.

1. For each of the following processes, where Z_t is white noise, express the model using the backward shift operator notation B and determine whether the process is stationary and/or invertible:

   (a) $Y_t = Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$

   (b) $Y_t = 0.5Y_{t-1} + Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$

   (c) $Y_t + 0.6Y_{t-1} = Z_t + 1.2Z_{t-1}$.

   Express each ARMA process as a general linear process.

2. For the MA(2) process
$$Y_t = Z_t + 0.5Z_{t-1} - 0.25Z_{t-2}$$
find the autocorrelation function. Is this process invertible?

3. Show that the AR(2) process
$$Y_t = Y_{t-1} + cY_{t-2} + Z_t$$
is stationary provided $-1 < c < 0$. Find the autocorrelation function when $c = -3/16$.

4. Show that the AR(3) process
$$Y_t = Y_{t-1} + cY_{t-2} - cY_{t-3} + Z_t$$
is non-stationary for all values of c.

5. Show that the autocorrelation function of the ARMA(1,1) process
$$Y_t = \alpha Y_{t-1} + Z_t + \beta Z_{t-1}$$
is given by
$$\begin{aligned} \rho_1 &= \frac{(1+\alpha\beta)(\alpha+\beta)}{(1+\beta^2+2\alpha\beta)} \\ \rho_k &= \alpha\rho_{k-1} \quad \text{for} \quad k = 2, 3, \ldots \end{aligned}$$

6. * Let $\{Y_t\}$ be the second-order autoregressive process defined by

$$Y_t = 0.3Y_{t-1} + 0.1Y_{t-2} + Z_t.$$

(a) Demonstrate that $\{Y_t\}$ is a stationary process, stating the rule for stationarity.

(b) Write down the Yule-Walker equations for this process.

(c) Show that the autocorrelation function of this AR(2) process is given by

$$\rho_k = 0.7619(0.5)^k + 0.2381(-0.2)^k,$$

for $k = 1, 2, \ldots$.

Patty Solomon
September 2004