Please note: Questions 1 and 5 (marked with asterisks) should be handed in as part of **Assignment 1**, *due by* **noon on Friday 27 August (week 5)**.

1. * *Covariances of linear combinations of random variables.* For constants a, b, c and d, show that

   (a) $\text{Cov}[a+X,\ Y] = \text{Cov}[X,\ Y]$

   (b) $\text{Cov}[aX,\ bY] = ab\text{Cov}[X,\ Y]$

   (c) $\text{Cov}[X,\ Y+Z] = \text{Cov}[X,\ Y] + \text{Cov}[X,\ Z]$

   (d) Hence show

   $$\text{Cov}[aW+bX,\ cY+dZ] = ac\text{Cov}[W,\ Y]+bc\text{Cov}[X,\ Y]+ad\text{Cov}[W,\ Z]+bd\text{Cov}[X,\ Z]$$

2. Suppose X and Y are random variables with means 3 and 5 respectively. If X and Y have variances 1 and 2 respectively and covariance 1, find

   (a) the correlation of X and Y

   (b) $\text{Var}[2X+3Y]$ and $\text{Var}[X-Y]$

   (c) $\text{Cov}[2X+3Y,\ X-Y]$

   (d) the correlation of $2X+3Y$ with $X-Y$

3. Assume random variables X and Y have variances σ_X^2 and σ_Y^2, covariance σ_{XY} and correlation ρ. Show that

   $$\text{Var}\left[\frac{X}{\sigma_X} \pm \frac{Y}{\sigma_Y}\right] = 2(1\pm\rho)$$

   and hence show $-1 \leq \rho \leq 1$.

4. Let $\{Z_t\}$ be a sequence of independent normal random variables, each with mean zero and variance σ^2, and let a, b and c be constants. Which, if any, of the following processes are stationary? For each stationary process specify the mean and autocovariance function.

   (a) $Y_t = a + bZ_t + cZ_{t-2}$

   (b) $Y_t = Z_1\cos(ct) + Z_2\sin(ct)$

   (c) $Y_t = Z_t\cos(ct) + Z_{t-1}\sin(ct)$

   (d) $Y_t = Z_tZ_{t-1}$

5. * A stationary random process $\{Y_t\}$ has mean μ and autocovariance function $\gamma_Y(k)$.

   (a) Show that the new process $\{D_t\}$, where $D_t = Y_t - Y_{t-1}$, is stationary and find its autocovariance function in terms of $\gamma_Y(k)$.

   (b) If Y_t defines a first order autoregressive process with parameter α, find an explicit expression for the autocovariance function of $\{D_t\}$ in terms of α.

6. Suppose we have a random sequence $\{U_t\}$ which is white noise. If we smooth the sequence using a moving average then remove the smoothed trend, this induces spurious autocorrelation into the residual series, as you will show.

   Define $R_t = U_t - S_t$, where

   $$S_t = (U_{t-1} + U_t + U_{t+1})/3$$

   is the simple moving average of order 3. Find the autocovariance function of $\{R_t\}$.

7. The first 10 sample autocorrelation coefficients of 400 'random' numbers are $r_1 = 0.02$, $r_2 = 0.05$, $r_3 = -0.09$, $r_4 = 0.08$, $r_5 = -0.02$, $r_6 = 0.00$, $r_7 = 0.12$, $r_8 = 0.06$, $r_9 = 0.02$ and $r_{10} = -0.08$. Is there any evidence of non-randomness?

Patty Solomon
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