[No. of pages: 6]
[No. of questions: 6]
[Total marks: 90]

THE UNIVERSITY OF ADELAIDE

SCHOOL OF APPLIED MATHEMATICS

LEVEL III STATISTICS

(5675) TIME SERIES
(STATS 3005)

NOVEMBER 2003

Time: 2 hours

[In addition, candidates are allowed ten minutes before the exam begins, to read the paper.]

Answer all questions.
Each question is worth 15 marks.
Pages 3 to 5 contain plots referred to in the written questions.
A Formula Sheet is attached on Page 6.
Calculators are permitted for simple numerical calculations.

1. (a) Explain briefly why time series require dedicated statistical methods for analysis.

   (b) Let $\{Y(t)\}$ be a random function. Define what is meant by the statement that $\{Y(t)\}$ is *second-order (or weakly) stationary.*

   [*Note: hereafter in this examination paper, we assume a stationary process is second-order stationary.*]

   (c) Let $\{Y_t\}$ be the simple random walk defined by

$$Y_t = Y_{t-1} + Z_t$$

   where $\{Z_t\}$ is a white noise sequence with mean zero and variance σ_Z^2, $Y_0 = 0$ and $t = 1, 2 \dots$.

   (i) Show that $\text{var}(Y_t) = t\sigma_Z^2$.

   (ii) Explain briefly why the result in (i) demonstrates that the simple random walk is non-stationary.

2. (a) Let $\{Z_t\}$ be a white noise sequence with variance σ^2, and let c be a constant. Find the mean and autocovariance function for the process

$$Y_t = Z_1 \cos(ct) + Z_2 \sin(ct)$$

   (b) Name and describe briefly two methods of *smoothing* an observed time series.

3. (a) Define the *sample autocorrelation coefficients*, r_k, for lags $k = 0, 1, \dots$.

   (b) What is the *correlogram*? How is the correlogram used to assess whether a series is consistent with white noise?

   (c) Figure 1 (top picture) shows a time series of monthly deaths in the United Kingdom attributed to bronchitis, emphysema and asthma from January 1974 to December 1979. Figure 1 (bottom picture) shows the associated correlogram, and Figure 2 shows the periodogram.

   (i) Describe the features of the observed series. Based on the information and plots you are given, is there evidence of non-stationarity? Justify your answer.

   (ii) Figure 2 shows the periodogram for the series after the mean and any linear trend have been removed. Interpret and explain the information given about the series in the periodogram.

4. A first-order moving average process, MA(1), is defined by

$$Y_t = Z_t + \beta Z_{t-1}$$

   where $|\beta| < 1$, and $\{Z_t\}$ is white noise with mean zero and variance σ_Z^2.

   (a) Find the mean and autocorrelation function of $\{Y_t\}$.

(b) Show that the normalized spectrum for the MA(1) process is

$$f^*(\omega) = 1 + \frac{2\beta}{(1+\beta^2)}\cos(\omega)$$

(c) Figure 3 (top plot) gives a realisation of $n = 200$ simulated observations from a first-order moving average process $Y_t = Z_t + 0.9Z_{t-1}$. Figure 3 also shows the correlogram and periodogram for the observed series, the theoretical autocorrelation function, and the spectrum.

Relate the form of the observed series to the theoretical properties of the underlying process.

5. Let $\{Y_t\}$ be the AR(2) process defined by

$$Y_t = \frac{1}{12}Y_{t-1} + \frac{1}{12}Y_{t-2} + Z_t$$

(a) Write down the Yule-Walker equations for this process, and find the auxiliary equation.

(b) Hence show that the autocorrelation function of this AR(2) process is given by

$$\rho_k = \frac{45}{77}\left(\frac{1}{3}\right)^k + \frac{32}{77}\left(-\frac{1}{4}\right)^k$$

for $k = 1, 2, \ldots$.

6. (a) An ARMA(1, 1) process is defined by

$$Y_t = 0.5Y_{t-1} + Z_t - 0.3Z_{t-1}$$

where $\{Z_t\}$ is white noise with zero mean and variance σ^2.

(i) Express the model using the backward shift operator B. Hence, or otherwise, determine whether the process is stationary and/or invertible.

(ii) Express Y_t as a general linear process.

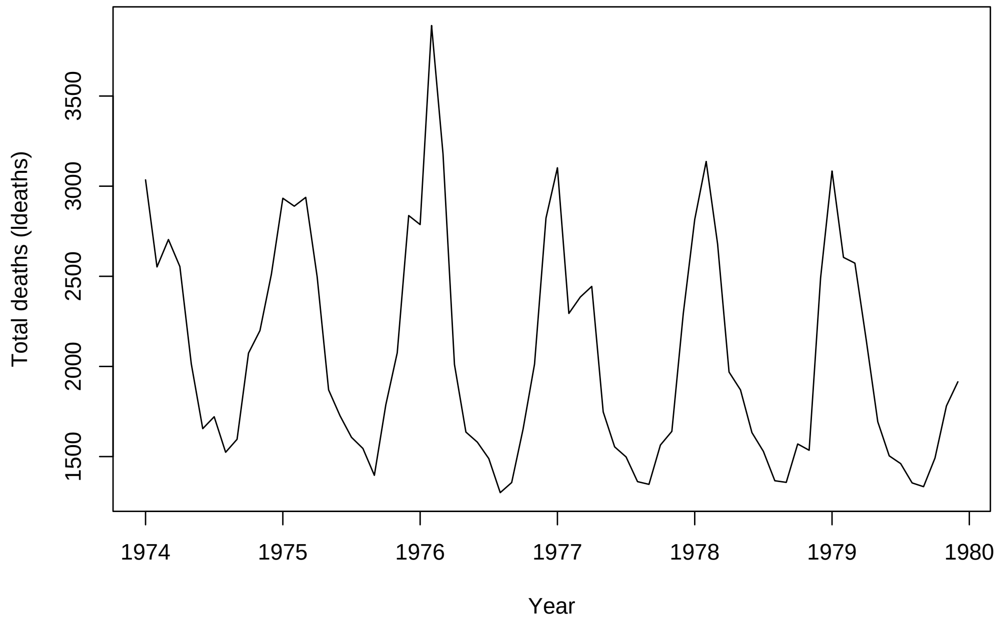
(b) Consider the ARIMA process defined by

$$(1 - 0.7B)(1 - B)Y_t = Z_t$$

(i) Identify p, d, q in this process.

(ii) Is the process stationary? Justify your answer.

Figure 1: Total UK deaths from common lung diseases



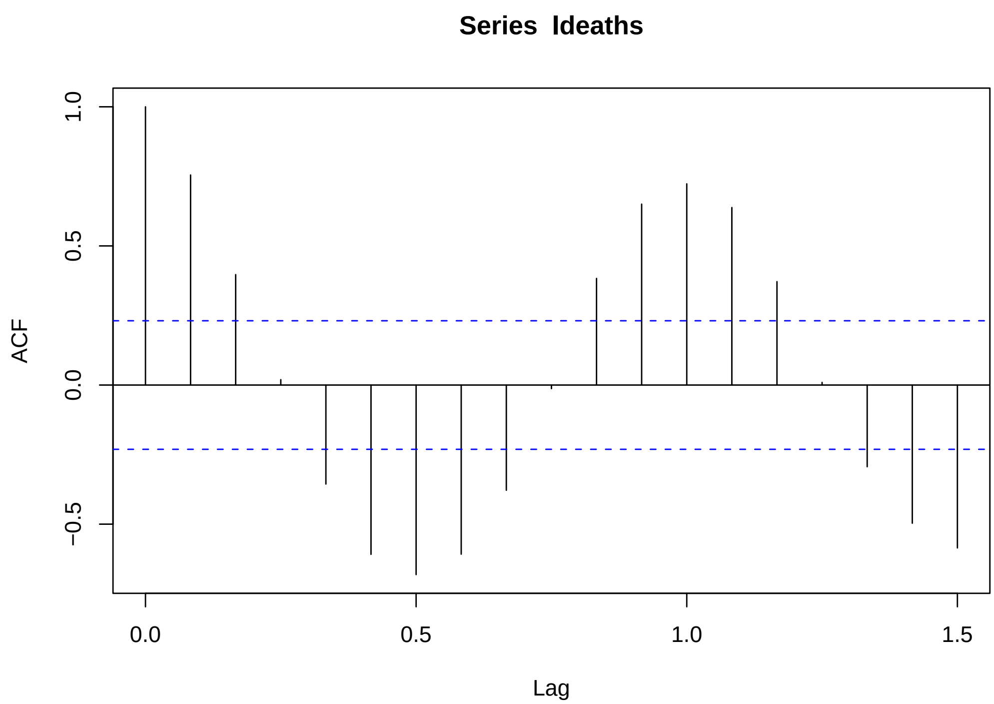


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Figure 2: Periodogram of series ldeaths

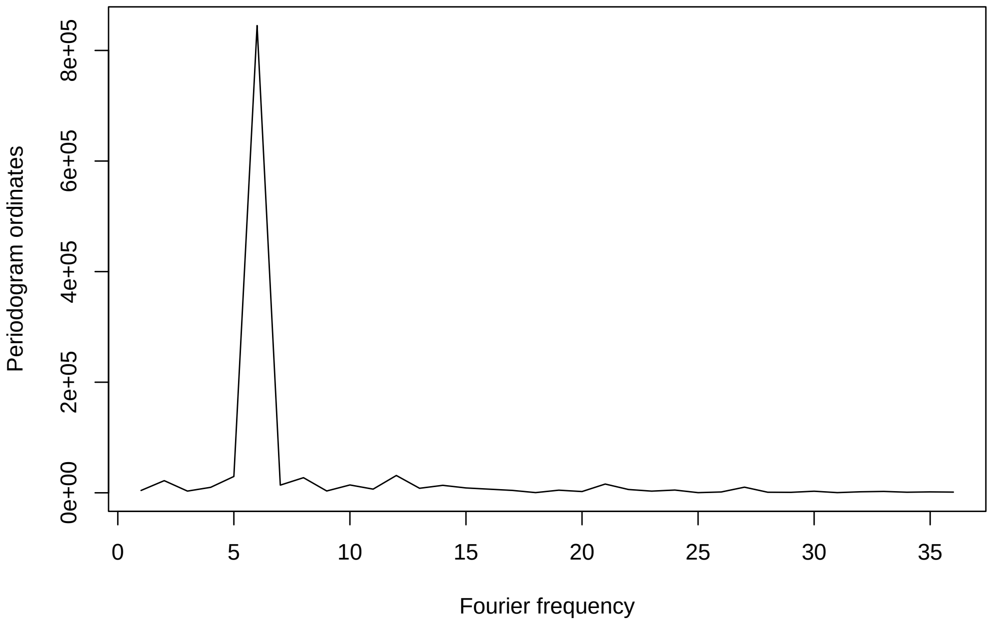

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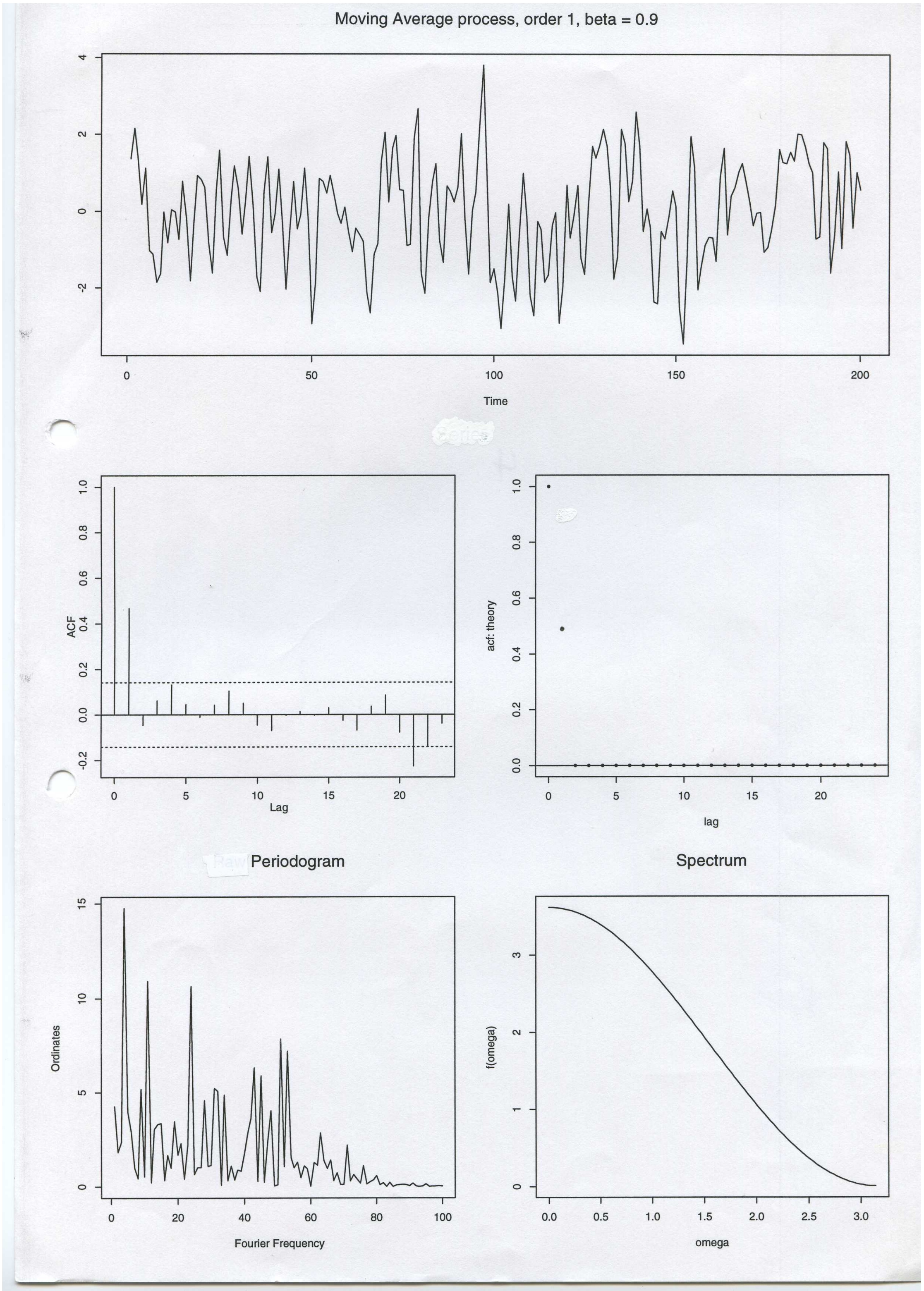
Figure 3: MA(1) process $Y_t = Z_t + 0.9Z_{t-1}$



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Formula Sheet for Time Series

Moving averages: A moving average of order $2p+1$ of a time series $\{y_t : t = 1, 2, \dots, n\}$ is a time series defined by

$$s_t = \sum_{j=-p}^{p} w_j y_{t+j}, \quad t = p+1, \dots, n-p$$

Sines and cosines:

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

Periodogram ordinates:

$$I(\omega) = \frac{1}{n} \left\{ \left(\sum_{t=1}^{n} y_t \cos(\omega t) \right)^2 + \left(\sum_{t=1}^{n} y_t \sin(\omega t) \right)^2 \right\}$$

where $0 < \omega \leq \pi$.

Yule-Walker equations for AR(p) process:

$$\rho_k = \sum_{j=1}^{p} \alpha_j \rho_{k-j}, \quad k = 1, 2, \dots$$

ARMA(p, q) process:

$$Y_t - \alpha_1 Y_{t-1} - \cdots - \alpha_p Y_{t-p} = Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q}$$

Spectrum and normalized spectrum:

$$f(\omega) = \gamma_0 + 2 \sum_{k=1}^{\infty} \gamma_k \cos(k\omega)$$

$$f^*(\omega) = 1 + 2 \sum_{k=1}^{\infty} \rho_k \cos(k\omega)$$