Time Series III

Overview of exam

- Assignment 3 due tomorrow
- recent lectures being added to My Uni
- course ‘ends’ with ARIMA
  (exclude p.a.c.f. onwards)

Today:
- exam format & outline
  → list non-examinable material
  → list formula to be provided in exam
  → coverage of questions
    inc. revision problems
Assessment:
Best 2 of 3 assignments
count 20%
+ 80% exam
or 100% exam

2003 Exam format:
• 2 hours
• 6 questions
  - approx. equal marks
  ~ 20 mins. per question

- Attempt all 6 questions
- Should be straightforward, drawn from lectures tutorials practicals assignment

(all familiar stuff)

Emphasis:
- definitions
- problem solving
- some derivation
- interpretation of practical work
A formula sheet will be provided in the exam:

- Moving average smoothing [2.8]
- Sine/cosine formulae
  
  \[ \cos(a-b) = \cos a \cos b + \sin a \sin b \]
  
  (See tut. 1, q4)

- Periodogram ordinates, \( I(\omega) \) [2.39]
- Yule-Walker equations [3.11]
- Full formula for ARMA \((p,q)\)

- Spectrum \( f(\omega) \) [4.4.4.5]
- Normalized spectrum \( f^*(\omega) \)

Topics definitely *not* on the exam: *third year*:

- Derivation of smoothing splines [2.14; 2.16]
- Variogram [2.31]
- Least squares derivation of p'gram ordinates [2.35, 2.38]
- 'R and spectral analysis' [Lecture 10]
- Connection b/w p'gram and c'gram [2.41; Ch.4]
• derivation of cumulative p'gram
  [2-42: 2-45; Lecture 12]

• transformations [2-52: 2-53]
  §2.8

• proof of Theorem for establishing stationarity of AR(p) [3-10; handout]

• derive Yule-Walker equations [3-12: 3-13]

  or complex roots case

• general derivation of acf for ARMA (1, 1) [Q5, Tut 3; Lecture 18]

• didn't cover [4-8: 4-13]

• partial acf [5-1: 5-3; Practical 5]

• [5.8 \rightarrow onwards]

Also not: (third years)

• Q.5*, Tut 1

• Q.2  Tut 2

• Q.3  "

• Q.7  " ←

• Q.5  Tut 3
Coverage of questions:

1. Key notions and quantities in time series.

2. Finding the acf; smoothing.

3. The correlogram and periodogram.


5. Application of Yule–Walker equations to find acf for AR(2). (Real, distinct roots.)

6. ARMA (1, 1) processes: stationarity; invertibility; as GLS.

ARIMA: identify p, d, q.
Question 1

- Why do time series require special methods for analysis?

- Be able to describe characteristics of t.s. [1-12]

- Be able to define trend in mean $\mu(t) = E[Y(t)]$

- Autocovariance function $\gamma(s,t) = \text{cov}(Y(s), Y(t))$
  
  $= E[(Y(s) - \mu(s))(Y(t) - \mu(t))]$

- Be able to define stationarity
  - We distinguish full (or strict) stationarity from second-order (or weak) stationarity.
  
  $\Rightarrow$ constant mean $\mu(t) = t$ and acf depends only on the lag $k$

  ie. $\gamma(s,t) = \gamma(k)$
  
  $= \text{cov}(Y(t), Y(t-k))$

  where $k = t-s$.

  This implies variance constant too.
- e.g.'s
  - Find acf for \( Z_t \) white noise
  
  We know \( Z_t \) has 0 mean, constant var. \( \sigma^2 \), and are independent, so \( Y_t = Z_t \)
  
  \( \gamma(s, t) = \text{cov}(Y_s, Y_t) = \text{EC}(Y_s Y_t) \)
  
  \( = \sigma^2 \) when \( s = t \)
  
  0 otherwise.

- Role of stationarity?
  Provides a degree of replication within a single t.s., which we

- Evidence of non-stationarity?
  - Seasonal effect
  - Increasing or decreasing trend
  - Non-constant variance

  E.g. simple random walk.
  Showed \( \text{var}(Y_t) = \sigma^2 t \), non-stationary.
  
  First differences stationary:
  
  \( Y_t - Y_{t-1} = Z_t \).

  As ARIMA: \( (1-B)Y_t = Z_t \)

  I.e. ARIMA(0, 1, 0).
Question 2

- be able to state 3 methods of smoothing: relative advantages & disadvantages
- know smoothing is an exploratory tool which represents data as
  \[ y_t = s_t + u_t \]
- be able to describe purposes of smoothing
  \[ s_t = \frac{2}{3} s_{t-1} + \frac{1}{3} y_t \]
- be able to define a 3-point moving average.

- be able to show removing low-order mov. av. induces autocorrelations in an observed series
  Candidate questions:
  - Tut 1, Q6. [2-32:2-3]
  - Pract. 2.
  - be able to find autocovariance and autocorrelation functions
    egs AR(1) (lectures)
    Tut. 1, Q4 (a), (b)
    Tut. 2, Q1.

and show stationary (or otherwise...
model as

\[ Y(t) = \mu(t) + U(t) \]

\[ \uparrow \]

stationary random fet, mean 0.

---

**Question 3:** mostly practical.

- be able to define sample autocorrelations, \( r_h \).
- know their properties under white noise hypothesis.
- for examples we have studied relate pattern in correlogram to theoretical acf's. e.g. AR(1) \( (p = \frac{n}{j}) \)
- be able to interpret periodogram of an observed series (i.e. identify and interpret the dominant peaks \( \rightarrow \) model.
• candidate examples:
  - Carbon dioxide data
  - UK Lung deaths data

• be able to discuss a simulated realisation from an AR(1), MA(1), ARMA(1,1) process, and relate to theoretical acf, spectrum and their estimates.

---

**Question 4**

• be able to define a linear filter, general linear process, AR(p), MA(q) etc. and write in BSO notation

• be able to find mean, variance and acf for MA(1) \([\text{Ass. 2; 3-8}]\)

• show MA(1) with $\beta, \frac{1}{\beta}$ have same acf.

• know that invertibility ensures a unique MA process
for a given acf. \(|\beta| < 1\).

- be able to establish invertibility.

- be able to find spectrum \(f(\omega)\) and normalized spectrum \(f^*(\omega)\) for \(MA(1)\).

**Question 5**

[See Ass. 3, (Tut 3, Q6*)] [3-14]

- be able to write \(AR(p)\) as GLP [see Ass. 2]

- be able to find \(\phi_k\) (acf) for an \(AR(2)\) process by solving the Yule-Walker equations. [real, distinct roots only]

- be able to establish whether \(AR(p)\) stationary.
Question 6

- be able to write ARMA processes using B notation, and as
  \[ \phi(B) Y_t = \theta(B) Z_t \]

- be able to establish whether ARMA\((p, q)\) is stationary and/or invertible \([\text{Tut 3, Q.1}]\)

- write \(\{Y_t\}\) as GLP by inverting \(\phi(B)\) for specific examples \([3-15], \text{L.18}, \text{Tut3, Q1}\).

- be able to identify \(p, d, q\) in an ARIMA model
e.g. last lecture, Diggle handout.