1.7 Multivariate distributions

Definition. 1.7.1

If \(X_1, X_2, \ldots, X_r\) are discrete RV’s then, \(X = (X_1, X_2, \ldots, X_r)^T\) is called a discrete random vector.

The probability function \(P(x)\) is:

\[
P(x) = P(X = x) = P \left( \{X_1 = x_1\} \cap \{X_2 = x_2\} \cap \cdots \cap \{X_r = x_r\} \right);
\]

\[
P(x_1, x_2, \ldots, x_r)
\]

joint prob.

1.7.1 Trinomial distribution

Consider a sequence of \(n\) independent trials where each trial produces:

- Outcome 1: with prob \(\pi_1\)
- Outcome 2: with prob \(\pi_2\)
- Outcome 3: with prob \(1 - \pi_1 - \pi_2\)

If \(X_1, X_2\) are number of occurrences of outcomes 1 and 2 respectively then\((X_1, X_2)\) have trinomial distribution.

Parameters: \(\pi_1 > 0, \pi_2 > 0\) and \(\pi_1 + \pi_2 < 1; n > 0\) fixed

Possible Values: integers \((x_1, x_2)\) s.t. \(x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq n\)

Probability function:

\[
P(x_1, x_2) = \frac{n!}{x_1!x_2!(n - x_1 - x_2)!} \pi_1^{x_1} \pi_2^{x_2} (1 - \pi_1 - \pi_2)^{n-x_1-x_2}
\]

for

\[x_1, x_2 \geq 0, \quad x_1 + x_2 \leq n.\]