School of Mathematical Sciences
INTRODUCTION TO MATHEMATICAL STATISTICS II
Semester 1, 2004

OUTLINE FOR WEEKS 7 AND 8

PREPARATION

This fortnight, we will finish Section 3 in the lecture notes, and start work on Section 4, which corresponds to Chapter 4 in WMS and deals with *continuous distributions*. You will need to revise integration, such as integration by parts and change of variable techniques, in order to deal with the integrals we shall meet in this section. For example:

$\int x^n dx = x^{n+1}/(n+1)$	$\int_0^\infty e^{-x} dx = 1$
For change of variable, where $w = g(z)$:	$\int f(w) dw = \int f\{g(z)\} g'(z) dz$
For integration by parts:	$\int_a^b f(t) g'(t) dt = [f(t)g(t)]_a^b - \int_a^b f'(t) g(t) dt$
$\Gamma(r) = \int_0^\infty t^{r-1} e^{-t} dt$	$\Gamma(r) = (r-1)!$ if r is integer

Lecture	Section	Overheads	Title	WMS	Pages
12	3.8	3-36:3-41	Normal distribution (revision)	4.5	170-175
	3.9	3-42:3-46	Normal approx. to binomial	7.5	354-360
	3.10	3-47:3-55	Poisson distribution	3.8	124-131
13	3.11	3-56:3-59	Moment generating functions	3.9	131-136
	3.12	3-60:3-64	Bounding probabilities	3.11	139-142
	3.13	3-65:3-68	Tchebychev's Inequality	"	"
14	4.1	4-1:4-3	Cumulative dsn functions	4.1	150-162
	4.2	4-4:4-9	Probability density functions	"	"
	4.3	4-10:4-12	Expected values	4.3	162-166
15	4.4	4-13:4-15	Uniform distribution	4.4	166-170
	4.5	4-16:4-22	Normal distribution	4.5	170-175
	4.6	4-23:4-25	Cauchy distribution	-	

TUTORIAL 4/ASSIGNMENT 2

This tutorial will be held in Week 8.

Starred questions (*) are part of Assignment 2 which is due by 10am on Friday 14 May (end Week 9). Note change of hand-in date.

1. *We will discuss first the Poisson questions (6. and 7.) and moment generating question (9.) from Tutorial 2.*

2. **Moment generating functions: this question will be discussed in the Tutorial.**

   If $W = aY + b$ for constants a and b, show that the moment generating function of W is $m_Y(at)e^{bt}$, where $m_Y(.)$ is the moment generating function of Y.

   Hence show that the mean and variance of W are given by
   $$\mu_W = a\mu_Y + b \quad \text{and} \quad \sigma_W^2 = a^2 \sigma_Y^2.$$

3. **To be done in groups in the Tutorial.**

   In a section of pine forest, the number of diseased trees per acre, Y, has a Poisson distribution with mean $\lambda = 10$. The diseases trees are sprayed with an insecticide at a cost of \$3.00 per tree, plus a fixed overhead cost for equipment rental of \$50.00. Let C denote the total spraying cost for a randomly selected acre.

   (a) Find the expected value and standard deviation of C.

   (b) Within what interval would you expect C to lie with probability at least 0.75?

4. * The mint produces 5c coins which have an average diameter of 12mm and a standard deviation of 0.2mm.

   (a) Find a lower bound for the proportion of coins that are expected to have diameters between 11.6 and 12.4mm.

   (b) Now suppose that the diameters are normally distributed. What is the probability that a randomly selected coin has a diameter inside the stated limits?

   (c) In a sample of 400, what is the distribution of the number of coins within the limits?

   What number would you expect to be within the limits?

5. (**This question will be discussed in the Tutorial.**)

   As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \frac{b}{y^2}, \quad y \geq b$$

   and zero elsewhere, where b is the minimum possible time needed to traverse the maze.

   (a) Show that $f(y)$ has the properties of a density function.

   (b) Find the cdf, $F(y)$.

   (c) Find $P(Y > b + c)$ for a positive constant c.

   (d) If c and d are both positive constants such that $d > c$, find $P(Y > b + d | Y > b + c)$.

   (e) Can you find the mean and variance of Y?

6. * Suppose we make parts on a production line and measure the amount Y by which the weight of the part differs from the target weight. From experience, we know that the distribution of Y is given by the density function

$$f(y) = c(1 - y^3), \quad -1 \leq y \leq 1$$

   where $f(y)$ takes value 0 elsewhere.

(a) Find the constant c.

(b) Find the cumulative distribution function $F(y)$.

(c) Find $P(Y < 0.5)$.

(d) Find the expected value of Y, μ_Y.

(e) Find the standard deviation of Y, σ_Y.

The following two questions will be discussed in the Tutorial if time permits.

7. Let U be a uniform random variable on the interval $(0, 1)$. Find the expected value of $Y = U^2$.

8. Let Y be uniformly distributed on the interval $(1, 2)$. Find $E(1/Y)$.

   Is $E(1/Y) = 1/E(Y)$? Justify your answer.

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