1. Suppose $X_1, \ldots, X_n$ are independent and identically distributed exponential random variables with mean $\beta$ and cumulative distribution function $F(X)$. Given the percentile points $p_{(1)}, \ldots, p_{(n)}$, where
$$p_{(i)} = \frac{i - 0.5}{n}, \quad i = 1, \ldots, n$$
find the form of the corresponding theoretical quantiles $x_{(i)} = F^{-1}(p_{(i)}), i = 1, \ldots, n$.

2. For events $A$ and $B$, prove Bonferroni’s inequality
$$P(A \cap B) \geq P(A) + P(B) - 1.$$ 

3. For events $A_1, \ldots, A_g$, prove that
$$P\left(\bigcup_{j=1}^{g} A_j\right) \leq \sum_{j=1}^{g} P(A_j).$$

4. * Suppose $g = 20,800$ genes are represented on a microarray, and that of these, 200 are truly differentially expressed. A single-sample $t$ statistic is obtained for each gene using $n$ replicated arrays, and a Type I error of 5% is used for each gene.
   (a) Calculate the approximate number of false positives produced.
   (b) The experiment-wise Type I error is chosen to be 5%. Using the Bonferroni method, find the Type I error for each of the 20,800 tests.
   (c) In principle, how could the problem of too many false negatives be overcome?

5. * Let $X_1, \ldots, X_g$ be i.i.d. continuous random variables with density function $f_X$ and cdf $F_X$.
   (a) Show that the minimum of $X_1, \ldots, X_g$, denoted $X_{(1)}$, has density function
   $$f_{X_{(1)}}(x) = g f_X(x)(1 - F_X(x))^{g-1}.$$ 
   (b) Now let the $X_1, \ldots, X_g$ be i.i.d. uniform $(0,1)$ random variables. Using part (a) or otherwise, find the density function for $X_{(1)}$. 
(c) For a given probability $\alpha$, find the value $K(g, \alpha)$ such that 

$$P(X_{(1)} \leq K(g, \alpha)) = \alpha.$$ 

6. Let $X$ be a continuous random variable with density function $f(x)$ and cdf $F(X)$, and assume $F^{-1}$ exists. Show that $P = F(X)$ has the uniform distribution on $(0, 1)$.

Questions marked with an asterisk * are part of Assignment 2. These questions are due by 4pm on Monday 23 September.