Biostatistics III

Lecture 9.

Reminder: Assignment 2 due end Week 6 (5/9).

Note: move Comp. Prac. 3 to Monday Week 7.

Now, for $\sigma^2, \alpha, \delta_0$ given, then $\beta$ is a function of $n$.

For trial/sample size calcs, we specify $\beta$, then solve for $n$. 

$H_0$ dsn for $\bar{x}_A - \bar{x}_B$.

$H_A$ dsn

$\beta = Type II$ error prob.

$= P(\text{retain } H_0)$

$H_A$

$\bar{x}_A - \bar{x}_B$

$\bar{x}_A - \bar{x}_B$

critical Value

$\bar{x}_A - \bar{x}_B$

$\bar{x}_A - \bar{x}_B$
Consider
\[ Z = \frac{\bar{X}_A - \bar{X}_B}{\sigma \sqrt{\frac{1}{n}}} \]

We can write this as
\[ = \left( \sum_{i=1}^{n} X_{Ai} - \sum_{i=1}^{n} X_{Bi} \right) \frac{1}{\sqrt{2n\sigma^2}} \]
\[ \sim N \left( \frac{(\mu_A - \mu_B)}{\sqrt{\frac{2\sigma^2}{n}}} , 1 \right) \]

Under \( H_0 \):
\[ Z \sim N(0, 1) \]

Under \( H_A \):
\[ Z \sim N \left( \pm \sigma_0 \sqrt{\frac{n}{2\sigma^2}} , 1 \right) \]

Consider positive alternative:
\[ \mu_A - \mu_B = \delta \]
and ignore very small prob. that \( Z < -z_\alpha \).

Thus
\[ \beta = P_{H_A} (Z - \delta \sigma_0 \sqrt{\frac{n}{2\sigma^2}} \leq -z_\alpha) \]
\[ \Rightarrow \beta = P_{H_A} (Z - \delta \sigma_0 \sqrt{\frac{n}{2\sigma^2}} \leq Z_\alpha - \delta \sigma_0 \sqrt{\frac{n}{2\sigma^2}}) \]
Rearranging,

\[ n = (Z_\alpha + Z_\beta)^2 \frac{2\sigma^2}{\delta_0^2} \]

To achieve power \( \geq 1 - \beta \)

(i.e. better separation)

we must have

\[ Z_\alpha - \delta_0 \sqrt{\frac{n}{2\delta^2}} \leq -Z_\beta \]

\[ \Rightarrow \quad n \geq (Z_\alpha + Z_\beta)^2 \frac{2\sigma^2}{\delta_0^2} \]
\( n \) is the minimum number of patients required in each group.

- We want high value for power, which can only be controlled at design stage.
  [Can control \( \alpha \) during analysis by choice of sig. level.]
- If \( \sigma \) estimated, can use \( t \)-distribution.

**Example:** we propose to study lung function in 2 groups of men. Response is FEV, forced expiratory volume.

From previous studies, we know \( \sigma = 0.5 \).

The minimum 'clinically significant' difference is \( \delta_0 = 0.25 \).

Use 2-sided test \( \alpha = 0.05 \), and power 80%. (usual power)

Q: how many men are required in each group?
\[ \alpha = 0.025, \quad Z_{\alpha} = 1.96 \]
\[ \beta = 0.2, \quad \text{so } Z_{\beta} = 0.842 \]

So,
\[ n \geq 2 \left\{ \frac{(1.96 + 0.842) \times 0.5}{0.25} \right\}^2 \]
\[ = 62.3. \]

i.e. minimum trial size that achieves 80% power is 63 men in each group.

If want 95% power?

**Exercise:** Need at least 104 men in each gp.