1. Show that the beta($\alpha, \beta$) family of distributions with density function
\[
f(y; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}, \quad 0 < y < 1, \quad \alpha > 0, \quad \beta > 0,
\]
with both $\alpha$ and $\beta$ unknown, is an exponential family. Describe the natural parameter space.

2. Let $Y_1, \ldots, Y_n$ be a random sample from a gamma($\alpha, \beta$) distribution with density function
\[
f(y; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y},
\]
where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter. Find a two-dimensional sufficient statistic for $(\alpha, \beta)$.

3. Suppose $Y_1, \ldots, Y_n$ is a random sample from a geometric distribution with probability function
\[
P(Y = y) = p(1-p)^{y-1}, \quad y = 1, 2, \ldots, \quad 0 < p < 1.
\]
Show that $\sum_{j=1}^{n} Y_j$ is sufficient for $p$. Find the distribution of $\sum_{j=1}^{n} Y_j$.

4. Let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. $N(\mu, 1)$. Show that $\bar{Y}$ is minimal sufficient for $\mu$.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from the location exponential distribution with density
\[
f(x; \theta) = e^{-(x-\theta)}, \quad \theta < x < \infty, \quad -\infty < \theta < \infty.
\]
Does this distribution belong to an exponential family of distributions? Find a minimal sufficient statistic for $\theta$.

6. Let $Y_1, \ldots, Y_n$ be independent and identically distributed gamma($\alpha, \alpha$) random variables with density
\[
f(y; \alpha, \alpha) = \frac{\alpha^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\alpha y}, \quad \alpha > 0, \quad x > 0.
\]
Show that $(\prod_{j=1}^{n} Y_j, \sum_{j=1}^{n} Y_j)$ is sufficient but not complete.

This assignment is due by 4pm on Wednesday 26 March.

Patty Solomon
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