Arrangement for tutorials in Week 9: As two of the tutes and one lecture are lost on the public holiday on the Monday 6th October I am cancelling the tutorial on the Friday 10th October. The lecture on the Thursday 9th will be held. As it happens I will also be unavailable that same week during my consulting hour. I will be available at other times, by email and I will be about for the two-week teaching break.

1. Define \( \cosh(z) = \frac{e^z + e^{-z}}{2} \) and \( \sinh(z) = \frac{e^z - e^{-z}}{2} \) and find formulae for them in terms of \( \sin, \cos, \sinh \) and \( \cosh \) of \( x \) and \( y \) where \( z = x + iy \).

2. Calculate \( a^b \) where
   (i) \( a = 1 + \sqrt{3}i, b = 1 - i \) (ii) \( a = 1 - i, b = i \) (iii) \( a = 23.14, b = i \).
   You will need a calculator for (iii) — can you give a reason for the form of the answer?

3. Let \( C \subset \mathbb{C} \) be the curve \( \{ (x, y) \mid y = 1 - x^2, y > 0 \} \) oriented so that it begins at \(-1\) and ends at \(1\). Compute:
   (i) \( \int_C zdz \) (ii) \( \int_C \sin(z)dz \) (iii) \( \int_C z\bar{z}dz \).

4. Let \( C \subset \mathbb{C} \) be the curve which joins 1 to \(-1\) by a straight line along the real-axis. Compute the three integrals in Question 3 again for this curve. Think before you do it. For which of the integrals did you need to do no work?

5. Let \( C \) be any curve from 1 to \( i \). Evaluate the following integrals:
   (i) \( \int_C \sin(z)dz \) (ii) \( \int_C (3z - 4i)^2dz \)

6. Use Cauchy’s integral formula and partial fractions to evaluate
   \[ \int_{|z|=2} \frac{dz}{z^2 + 1} \]
   where the curve is oriented in an anti-clockwise direction.

7*. A function \( f : U \rightarrow \mathbb{R} \) on an open subset of \( \mathbb{R}^2 \) is called harmonic if it satisfies the partial differential equation
   \[ f_{xx} + f_{yy} = 0. \]
   \[ (f_{kk} = \frac{\partial^2 f}{\partial x^2} \text{ etc}) \]
   Show that if \( f = u + iv \) is complex analytic then \( u \) and \( v \) are harmonic.