1. Recall that cos(z) and sin(z) are defined as:

\[\cos(z) = \frac{e^{iz} + e^{-iz}}{2}\quad\text{and}\quad\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}\.\]

Using this show that:

(i) \(\cos^2(z) + \sin^2(z) = 1\)
(ii) \(\frac{d}{dz}\cos(z) = -\sin(z)\)
(iii) \(\frac{d}{dz}\sin(z) = \cos(z)\).

2. Calculate \(a^b\) where

(i) \(a = \sqrt{3} + i, b = i\)
(ii) \(a = i, b = i\)
(iii) \(a = 6, b = 2i\).

3. Let \(C \subset \mathbb{C}\) be the curve \(\{(x, y) | x^2 + y^2 = 1, y > 0\}\) oriented so that it begins at 1 and ends at \(-1\). Compute:

(i) \(\int_C \Re(z)dz\)
(ii) \(\int_C \sin(z)dz\)
(iii) \(\int_C z\bar{z}dz\).

4. Let \(C\) be the circle of radius 1 about 0 in \(\mathbb{C}\) oriented in an anti-clockwise direction. If \(z = x + iy\) show that

\(\int_C x^2 - y^2 dz = -i\int_C 2xydz\).

[Hint: You do not need to calculate.]

5. Let \(C\) be any curve from 2 to \(-i\). Evaluate the following integrals:

(i) \(\int_C \sin^2(z)\cos(z)dz\)
(ii) \(\int_C (z - i)^2dz\)
(iii) \(\int_C z\exp(z^2)dz\).

6. Use Cauchy's integral formula and partial fractions to evaluate the following integrals where both curves are oriented in an anti-clockwise direction:

(i) \(\int_{|z|=10} \frac{1}{z^2 + (2 + i)z + 2i}dz\)
(ii) \(\int_{|z|=5} \frac{\sin(z)}{z^2 - 3}dz\)

7*. Consider the integral

\(\int_C \frac{1}{z - 1}dz\)

where \(C\) is a curve joining 0 to 2. If we consider different choices of curve how many different values can this integral take? Calculate the different possible values.

It's difficult to rigourously prove the answer for this question — so it's really just one for you to think about during the teaching break. Thinking about these questions was the beginning of the subject called topology.