9. CALCULUS OF MORE THAN ONE VARIABLE

Review

Recall that last lecture we began to look at functions of of more than one variable.

9.2 Functions of Several Variables

The main features of single-variable calculus — limits, derivatives, chain rule, maximum-minimum techniques — all generalise to functions of several variables.

Definition. Let $\mathcal{D}$ be a subset of $\mathbb{R}^2$. Suppose there is a relation which assigns to each (x, y) in $\mathcal{D}$ a real number $f(x, y)$. Then f is said to be a function of two variables with domain $\mathcal{D}$.

Definition. Let f be a function of two variables with domain $\mathcal{D}$. The surface consisting of all points (x, y, z) of $\mathbb{R}^3$ such that

$$z = f(x, y)$$

is called the graph of f.

Contours and level curves

An alternative method is often used to represent the graph of a function of 2 or more variables; namely to use contours or level curves.

Definition. The intersection of the horizontal plane $z = k$ with the surface $z = f(x, y)$ is called the contour curve of height k on the surface.

The vertical projection of this contour curve onto the xy-plane is called the level curve $f(x, y) = k$ of function f. Thus the level curves of f are curves in the xy-plane on which the value of f is constant.