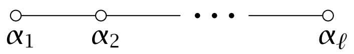
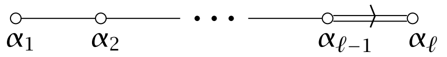
Lie Algebras IV 2009

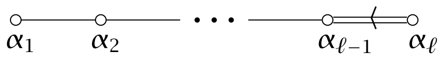
Dynkin Diagrams



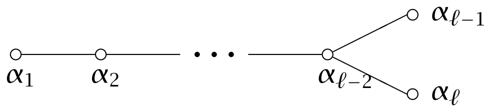
(type A_ℓ, $\ell \geq 1$)



(type B_ℓ, $\ell \geq 2$)

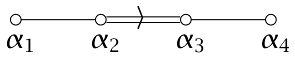


(type C_ℓ, $\ell \geq 3$)

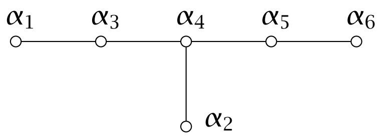


(type D_ℓ, $\ell \geq 4$)

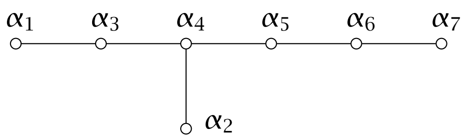
(type G_2)



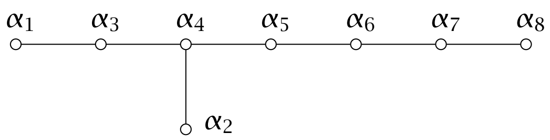
(type F_4)



(type E_6)



(type E_7)



(type E_8)

Table 1. *Cartan matrices*

$$A_\ell: \begin{pmatrix} 2 & -1 & 0 & & & \cdot & \cdot & \cdot & & 0 \\ -1 & 2 & -1 & 0 & & \cdot & \cdot & \cdot & & 0 \\ 0 & -1 & 2 & -1 & 0 & \cdot & \cdot & \cdot & & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & & \cdot & \cdot & \cdot & -1 & 2 \end{pmatrix}$$

$$B_\ell: \begin{pmatrix} 2 & -1 & 0 & & \cdot & \cdot & \cdot & & & 0 \\ -1 & 2 & -1 & 0 & \cdot & \cdot & \cdot & & & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & & \cdot & \cdot & \cdot & -1 & 2 & -2 \\ 0 & 0 & 0 & & \cdot & \cdot & \cdot & 0 & -1 & 2 \end{pmatrix}$$

$$C_\ell: \begin{pmatrix} 2 & -1 & 0 & & \cdot & \cdot & \cdot & & & 0 \\ -1 & 2 & -1 & & \cdot & \cdot & \cdot & & & 0 \\ 0 & -1 & 2 & -1 & \cdot & \cdot & \cdot & & & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & & \cdot & \cdot & \cdot & -1 & 2 & -1 \\ 0 & 0 & & & \cdot & \cdot & \cdot & 0 & -2 & 2 \end{pmatrix}$$

$$D_\ell: \begin{pmatrix} 2 & -1 & 0 & & \cdot & \cdot & \cdot & & & 0 \\ -1 & 2 & -1 & & \cdot & \cdot & \cdot & & & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & & \cdot & \cdot & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & & \cdot & \cdot & & -1 & 2 & -1 & -1 \\ 0 & 0 & & \cdot & \cdot & & 0 & -1 & 2 & 0 \\ 0 & 0 & & \cdot & \cdot & & 0 & -1 & 0 & 2 \end{pmatrix}$$

$$E_6: \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$E_7: \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$E_8: \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$F_4: \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -2 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \qquad G_2: \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$