1. Consider the Lie algebra $\mathfrak{gl}_n(C)$.
   
   a) Let $E_{ij}$ be the matrix with a one in the $(i, j)$ position and zeros elsewhere. Show that
   
   $[E_{ij}, E_{kl}] = \delta_{jk}E_{il} - \delta_{li}E_{kj}$
   
   where $\delta_{ij}$ is the Kronecker delta symbol which is one if $i = j$ and zero otherwise.

   b) Using part a) or otherwise find the derived algebra and centre of $\mathfrak{gl}_n(C)$. You might find it useful to compute things like: $[X, E_{ij}]$, for a general matrix $X$ and $[E_{kk}, E_{ij}]$ and $[E_{ij}, E_{ji}]$.

   c) Show that the upper triangular matrices in $\mathfrak{gl}_n(C)$ are a Lie subalgebra. Are they an ideal?

2. If $L$ is a finite-dimensional Lie algebra with basis $v_1, \ldots, v_r$ show that $\{ [v_i, v_j] \mid 1 \leq i < j \leq n \}$ is a spanning set for $L'$.

3. Let $\phi: L \to J$ be an isomorphism of Lie algebras. Show that $Z(J) = \phi(Z(L))$ and $J' = \phi(L')$.

4. Let $L = L_1 \oplus L_2$ where $L_1$ and $L_2$ are Lie algebras. Show that $L' = L'_1 \oplus L'_2$ and $Z(L) = Z(L_1) \oplus Z(L_2)$.

5. Let $L$ be a two-dimensional vector space with basis $x$ and $y$. Show that requiring bilinearity and antisymmetry and defining the bracket of $x$ and $y$ by $[x, y] = x$ suffices to define a unique Lie algebra $L$.

6. Show that $\mathbb{C}^3$ with the cross (vector) product as bracket is a Lie algebra. Calculate its centre and derived algebra. Find an explicit Lie algebra isomorphism between $\mathbb{C}^3$ and $\mathfrak{sl}(2, \mathbb{C})$. 