If \( X: V \to V \) is a linear map on a finite dimensional vector space \( V \) and \( V \) has a basis \( v^1, \ldots, v^n \) then we can find a matrix \( [X] \) which represents \( X \). But what is it? We would like to have that if \( Y: V \to V \) also then \((YX): V \to V \) has matrix \([YX] = [Y][X]\). In fancy language we want the map 
\[
gl(V) \to gl(n, \mathbb{C}) \\
X \mapsto [X]
\]
to be an algebra isomorphism.

To obtain this we need to define \([X]\) by 
\[
X(v^i) = \sum_{j=1}^{n} X_{ji} v^j
\]
which is not the obvious thing you might write down. But it works because
\[
\sum_{k=1}^{n} [YX]_{ki} v^k = (YX)(v^i)
\]
\[
= Y(X(v^i))
\]
\[
= Y\left(\sum_{j=1}^{n} X_{ji} v^j\right)
\]
\[
= \sum_{j=1}^{n} X_{ji} Y(v^j)
\]
\[
= \sum_{j=1}^{n} X_{ji} \left(\sum_{k=1}^{n} Y_{kj} v^k\right)
\]
\[
= \sum_{k=1}^{n} \left(\sum_{j=1}^{n} Y_{kj} X_{ji}\right) v^k
\]
\[
= \sum_{k=1}^{n} ([Y][X])_{ki} v^k
\]

As another test this is a good idea consider the action of a matrix \( X \) on column vectors. This defines a linear map \( X: \mathbb{C}^n \to \mathbb{C}^n \). We would rather hope that if we expand this linear map in the usual basis \( \{e^1, \ldots, e^n\} \) using the definition above that we find that the \( ij \)th element is \( X_{ij} \). To see this note that if I apply the matrix \( X \) to the vector \( e^i \) in the standard way it picks out the \( i \)th column of \( X \) which is also \( \sum_{j=1}^{n} X_{ji} e^j \). So \( X(e^i) = \sum_{j=1}^{n} X_{ji} e^j \) as we wanted.