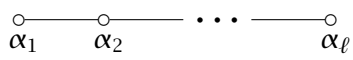
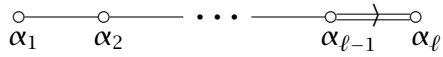


Lie Algebras IV 2008

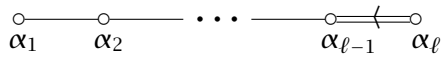
Dynkin Diagrams



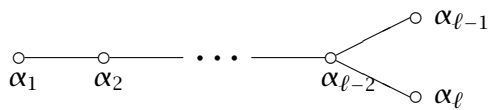
(type A_ℓ , $\ell \geq 1$)



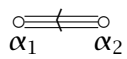
(type B_ℓ , $\ell \geq 2$)



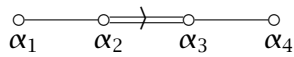
(type C_ℓ , $\ell \geq 3$)



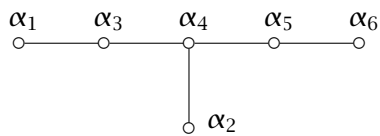
(type D_ℓ , $\ell \geq 4$)



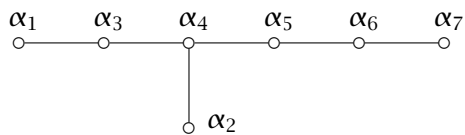
(type G_2)



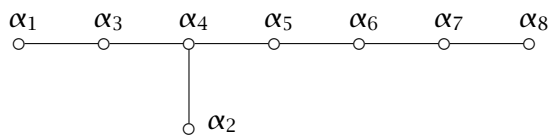
(type F_4)



(type E_6)



(type E_7)



(type E_8)

Table 1. Cartan matrices

$$A_4: \begin{pmatrix} 2 & -1 & 0 & & & & & 0 \\ -1 & 2 & -1 & 0 & & & & 0 \\ 0 & -1 & 2 & -1 & 0 & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & & & -1 & 2 \end{pmatrix}$$

$$B_4: \begin{pmatrix} 2 & -1 & 0 & & & & & 0 \\ -1 & 2 & -1 & 0 & & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & & & & -1 & 2 \\ 0 & 0 & 0 & & & & 0 & -1 \\ & & & & & & & & -2 \\ & & & & & & & & & 2 \end{pmatrix}$$

$$C_4: \begin{pmatrix} 2 & -1 & 0 & & & & & 0 \\ -1 & 2 & -1 & 0 & & & & 0 \\ 0 & -1 & 2 & -1 & & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & & & & -1 & 2 \\ 0 & 0 & & & & & 0 & -2 \\ & & & & & & & & -1 \\ & & & & & & & & & 2 \end{pmatrix}$$

$$D_4: \begin{pmatrix} 2 & -1 & 0 & & & & & 0 \\ -1 & 2 & -1 & & & & & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & & & -1 & 2 & -1 & 0 \\ 0 & 0 & & & & -1 & 2 & -1 \\ 0 & 0 & & & & 0 & -1 & 2 \\ 0 & 0 & & & & 0 & -1 & 0 \\ & & & & & & & & 0 \\ & & & & & & & & & 2 \end{pmatrix}$$

$$E_6: \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$E_7: \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$E_8: \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$F_4: \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -2 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$G_2: \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

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