Algorithm for computing $T, N, B, \kappa$ and $\tau$.

Let $y: (a, b) \rightarrow C$ be a parametrised curve in $\mathbb{R}^3$. To compute $T, N, B, \kappa$ and $\tau$ we proceed as follows.

First notice that if $X: C \rightarrow \mathbb{R}^3$ is any of $T, N$ or $B$ or even a general function the relation between differentiating with respect to arc-length and differentiating with respect to the $y$ parametrisation is given by

$$\dot{X} = \frac{X'}{\|y'\|}.$$

1. Calculate $y'(t)$ and $\|y'(t)\|$. Then

$$T = \dot{y} = \frac{y'}{\|y'\|}.$$

2. Calculate $T'$ and $\|T'\|$ then

$$N = \frac{T'}{\|T'\|}.$$

3. Using the result above

$$\dot{T} = \frac{T'}{\|y'\|}.$$

4. Deduce $\kappa$ from

$$\dot{T} = \kappa N.$$

5. Calculate $B$ from

$$B = T \times N.$$

6. Compute $B'$ and

$$\dot{B} = \frac{B'}{\|y'\|}.$$

7. Deduce $\tau$ from

$$\dot{B} = -\tau N.$$

All the answers above will be functions of $t$ and represent $T, N, B, \kappa$ and $\tau$ evaluated at $y(t)$.

Notice that you could use the formula in the notes to compute $\kappa$ but the method above is probably easier if you have to compute $T, N, B$ and $\tau$ as well. It is also apparent from the above that

$$\kappa = \frac{\|T'\|}{\|y'\|}.$$

This looks like it is an easy way to get $\kappa$ but in fact it requires you to calculate

$$\kappa = \frac{\left(\frac{y'}{\|y'\|}\right)'}{\|y'\|},$$

which comes out the same as the formula we derived in class.