Algorithm for computing T, N, B, κ and τ.

Let $\gamma\colon (a,b) \to C$ be a parametrised curve in $\mathbb{R}^3$. To compute T, N, B, κ and τ we proceed as follows.

First notice that if $X\colon C \to \mathbb{R}^3$ is any of T, N or B or even a general function the relation between differentiating with respect to arc-length and differentiating with respect to the γ parametrisation is given by

$$\dot{X} = \frac{X'}{\|\gamma'\|}.$$

1. Calculate $\gamma'(t)$ and $\|\gamma'(t)\|$. Then
$$T = \dot{\gamma} = \frac{\gamma'}{\|\gamma'\|}$$
2. Calculate T' and $\|T'\|$ then
$$N = \frac{T'}{\|T'\|}$$
3. Using the result above
$$\dot{T} = \frac{T'}{\|\gamma'\|}.$$
4. Deduce κ from
$$\dot{T} = \kappa N.$$
5. Calculate B from
$$B = T \times N.$$
6. Compute B' and
$$\dot{B} = \frac{B'}{\|\gamma'\|}.$$
7. Deduce τ from
$$\dot{B} = -\tau N.$$

All the answers above will be functions of t and represent T, N, B, κ and τ evaluated at $\gamma(t)$.

Notice that you could use the formula in the notes to compute κ but the method above is probably easier if you have to compute T, N, B and τ as well. It is also apparent from the above that

$$\kappa = \frac{\|T'\|}{\|\gamma'\|}$$

This looks like it is an easy way to get κ but in fact it requires you to calculate

$$\kappa = \frac{\left\|\left(\frac{\gamma'}{\|\gamma'\|}\right)'\right\|}{\|\gamma'\|}$$

which comes out the same as the formula we derived in class.