Revision lecture

(1) Discuss info sheet. ??

~~(2)~~

(2) Look at Mock Exam. / Solution.

(3) **Course outline:**

(1) Review: sequences, limits.

— not going to be examined

(2) **Def's.** Differentiable.

C^k. C^∞

Chain rule / Inverse / Implicit function theorems.

Expect you to know but not examining proofs.

No question like "show this is differentiable

~~(3)~~ calculate $f'(x)$" etc"!

BUT I ASSUME YOU CAN CALCUTE $f'(x)$!

~~3~~ (3) **SUBMANIFOLDS**

Def^n

Different conditions — defining equation, graph, parametrisation

Tangent space (important)

$\frac{\partial \psi}{\partial x^1}, \ldots, \frac{\partial \psi}{\partial x^n}$ basis.

function on ~~tangent space~~ submanifold $f : S \to \mathbb{R}^m$

Defn : $f'(x) : T_x S \to \mathbb{R}^m$

~~(Should be in an article 3.15)~~

Chain rule for function on submanifolds

④ CURVES

1-dim^l submanifold, one-one parametrisation for whole curve = param

Parametrisation by ARC-LENGTH

$\| \gamma'(t) \| = 1$ ~~for notation 4.3.~~

Curvature κ

T, N, B

τ.

Frenet equations – I'll give these.

5. SURFACES

Unit normal n.

$n : \Sigma \longrightarrow S^2$ Gauss map.

Second fundamental form

$\pi = -n' = -dn$.

Principal curvatures, mean & Gaussian curvature.

6. INTEGRATION

Def. Discussion of integration in $\mathbb{R}^n$.

Fubini's Thm.

~~Volume~~ Forms — n-form $\det(V^*)$ — 1 dim'l

$[v^1, \ldots, v^n] \in \det(V^*)$

Define

$$\psi : \underset{\mathbb{R}^n}{\overset{\cup\!\!\!\mid}{\mathcal{U}}} \longrightarrow \Sigma \qquad \operatorname{supp} \omega \subseteq \psi(\mathcal{U})$$

$$\int_{\psi(\mathcal{U})} \omega = I_\psi(\omega) = \int_{\mathcal{U}} \omega(\psi(x))\left(\frac{\partial \psi}{\partial x^1}(x), \ldots, \frac{\partial \psi}{\partial x^n}(x)\right) dx^1 \ldots dx^n$$

Partition of unity .... integral in general

Volume form Σ a surface

$$\operatorname{vol}(v, w) = \langle v \times w, n \rangle$$

$Rvd(v,w) = \langle \pi(v) \times \pi(w), n \rangle$

one-form. – dual basis etc

wedge product

$d\varphi^i$ ~ dual to $\frac{\partial \psi}{\partial x^i}$.

$\{ d\varphi^1 \wedge d\varphi^2 \} = \left[\frac{\partial \psi}{\partial x^1}, \frac{\partial \psi}{\partial x^2} \right]$

d(one-form) – def^n

$d(f\alpha) = df\, \alpha + f\, d\alpha$

weak Green's Th^m $\int d\alpha = 0$

α one-form

7. GAUSS-BONNET THEOREM

$\frac{d}{dt} \int Rvd = 0$

$\tilde{\Sigma} = \Sigma$ + handle

$\Rightarrow \int_{\tilde{\Sigma}} Rvd = \int_{\Sigma} Rvd - 2$

$\therefore \Sigma_g = S^2 + g$ handle $\int_{\Sigma_g} Rvd = 2 - 2g$

Tessellation

$$\chi = v - e + f$$

$$\chi(\Sigma_g) = 2 - 2g = \int_{\Sigma_g} K \, v \, d \qquad \text{GB Theorem}$$