THE UNIVERSITY *of* ADELAIDE

Examination in School of Mathematical Sciences

Semester 2 2011

105919	**Geometry of Surfaces III – MOCK EXAM**
	PURE MTH 3022

Official Reading Time:	10 mins
Writing Time:	120 mins
Total Duration:	130 mins

NUMBER OF QUESTIONS: 5 TOTAL MARKS: 70

Instructions

- Attempt all questions.
- Approximate marks are indicated.
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.

Materials

- 1 Blue book is provided.
- Calculators are not permitted.

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO.

1. For each of the following statements say whether it is True or False. Give a very short justification or counter-example in each case.

   (a) The curve $\gamma(t) = (t, t^2, t^{99})$ is arc-length parametrised.

   (b) A curve with constant unit tangent T has zero curvature κ.

   (c) It is possible to find a tessellation of the torus with 100 vertices, 92 edges and 76 faces.

   (d) A cylinder has Gaussian curvature which is everywhere positive.

   (e) There exists a surface with principal curvatures $\lambda_1 > 0$ and $\lambda_2 < 0$ and Gaussian curvature zero.

   [10 marks]

2. (a) Define the tangent space, T_sS to a submanifold $S \subset \mathbb{R}^N$ at a point $s \in S$.

   (b) Assume that $\psi\colon U \to S$ is a parametrisation of S where U is open in $\mathbb{R}^n$. Show that $T_sS = \operatorname{im}\psi'(x)$ if $s = \psi(x)$. You may assume that $\dim(T_sS) = n$.

   (c) Consider the submanifold defined by the equation $x^2 + y^2 = z$ in $\mathbb{R}^3$. Find the tangent space at a general point.

   [10 marks]

3. (a) Let $\gamma\colon (a, b) \to \mathbb{R}^3$ be a parametrised curve. Define T, N and B.

   (b) Consider the parametrised curve C with parametrisation $\gamma(t) = (t, t, \frac{t^2}{2})$.

      (i) Is γ an arc-length parametrisation of C? Justify your answer.

      (ii) Find T, N and B for C.

      (iii) Compute the curvature κ and the torsion τ of the curve C.

   You may assume the Frenet Formulae: $\dot{T} = \kappa N$, $\dot{N} = -\kappa T + \tau B$ and $\dot{B} = -\tau N$.

   [20 marks]

4. Consider the paraboloid P defined by the equation $x^2 + y^2 = z$.

   (a) Find a parametrisation whose image is all of P (do not prove it is a parametrisation just write it down).

   (b) Compute the normal to P oriented to that it points up at $(0, 0, 0)$.

   (c) Compute the Mean and Gaussian curvature of P.

   [15 marks]

Please turn over for page 3

5. (a) (i) Define what the volume form vol_S of an n dimensional oriented submanifold S is.

   (ii) If $f\colon S \to \mathbb{R}$ is a smooth function explain how you would integrate $f\,\mathrm{vol}_S$ over the image of an oriented parametrisation $\psi\colon U \to S$ where $U \subset \mathbb{R}^n$ is open.

   (b) Consider the surface T in $\mathbb{R}^3$ defined by the equation

$$(\sqrt{x^2+y^2} - R)^2 + z^2 = r^2$$

   for $0 < r < R$.

   (i) Sketch this surface, draw a tessellation on it and compute its Euler characteristic.

   (ii) If R is the Gaussian curvature of this surface what does the Gauss Bonnet theorem predict that $\frac{1}{2\pi}\int_T R\,\mathrm{vol}_T$ is ? (Don't try and compute this integral.)

[15 marks]