We have \( G(x,y,z) = x^2 + y^2 - z^2 - 1 = 0 \)

Tangent space at \((x,y,z)\) is \( \{ (\alpha,\beta,\delta) \mid x\alpha + y\beta - z\delta = 0 \} \)

Basis: \( (y, -x, 0), (0, z, y) \) \( \iff y \neq 0 \)

Element in \( (x,y,z) + T(x,y,z) \) is \( (x,y,z) + \lambda(y,-x,0) + \mu(0,z,y) \).

Set \( 0 = G(x+\lambda y, y-x\lambda + \mu z, z+\mu y) \)

\[ = x^2(y^2 + x^2) + \lambda y (-2x^2) + \mu z^2(y^2) \]

Homogeneous quadratic polynomial. If \( \lambda, \mu \)

is a solution so also is \( (t\lambda, t\mu) \) \( \forall \epsilon \mathbb{R} \) so

we get a line.

Discriminant = "\( b^2 - 4ac \)" = \( 4x^2z^2 - 4(y^2+x^2)(z^2-y^2) \)

\[ = 4y^2 > 0. \]

.: Two real solutions.: Two lines.

(If you prefer divide \( \lambda \) or \( \mu \) and get a

quadratic in \( \frac{\lambda}{\mu} \) or \( \frac{\mu}{\lambda} \).

If \( y=0 \) choose basis:

\( (y, -x, 0) \quad (z, 0, x) \quad (x \neq 0) \)

 cannot have \( x \) \& \( y = 0 \).

NB: This is a better approach for the handout
question.