We have $G(x,y,z) = x^2+y^2-z^2-1 = 0$

Tangent space at $(x,y,z) = \{(\alpha,\beta,\gamma) \mid x\alpha + y\beta - z\gamma = 0\}$

Basis : $(y,-x,0)$ $(0,z,y)$ at $y \neq 0$

Element in $(x,y,z) + T_{(x,y,z)}S$ is

$$(x,y,z) + \lambda(y,-x,0) + \mu(0,z,y).$$

Set
$$0 = G(x+\lambda y,\ y-x\lambda+\mu z,\ z+\mu y)$$
$$= \lambda^2(y^2+x^2) + \lambda\mu(-2xz) + \mu^2(z^2-y^2).$$

Homogeneous quadratic polynomial. If (λ,μ) is a sol^n so ~~also~~ also is $(t\lambda, t\mu)$ $\forall t \in \mathbb{R}$ so we get a line.

Discriminant = "b^2-4ac" $= 4x^2z^2 - 4(y^2+x^2)(z^2-y^2)$
$= 4y^2 > 0.$

$\therefore$ Two real solutions. $\therefore$ Two lines.

(If you prefer divide out λ or μ and get a quadratic in λ/μ or μ/λ.)

If $y = 0$ ~~so~~ choose basis
$(y,-x,0)$ $(z,0,x)$ $(x \neq 0)$

Repeat.

Cannot have x & $y = 0$.

NB: This is a better approach for the handup question.