School of Mathematical Sciences
PURE MTH 3007
Groups and Rings III, Semester 1, 2010

Brief comments on the 2009 examination

NB: These are brief comments NOT examples of the kinds of solutions I expect you to write !

Q1. (i) T, (ii) F (iii) T (iv) T (v) F (vi) F (vii) T (viii) T

Q2. (a) Lectures (b) Lectures (c) Lectures (d) From Lagrange $|Z(G)|$ is 1, p, q or pq. As G is not abelian $|Z(G)| \neq pq$. If $|Z(G)| = p$ then $|G/Z(G)| = q$ so $G/Z(G)$ is cyclic so G is abelian a contradiction. Similar if $|Z(G)| = q$ so $|Z(G)| = 1$. If G is abelian then it is finitely generated and abelian so isomorphic to $C_p \times C_q \cong C_{pq}$.

Q3. (a) Torsion invariants 3 and 6, rank is 1. (b) I'll list the torsion invariants you can deduce the decomposition into cyclic groups. $10, 30$. $5, 60$. $2, 150$. 300.

Q4. (a) Lectures (b) CE 4, Q 5

Q5 (a) Lectures (b) Lectures (c) Lectures. (d) $|G| = 7 \times 11$ and 11 is not congruent to 1 modulo 7 so proceed as in lectures.

Q6. (a) Lectures (b) $\mathbb{Z}_6$ and $\mathbb{Z}$. (c) Lectures. (d) Lectures

Q7. (a) Lectures and then note that for a unit we must have $a^2 + 3b^2 = 1$ whose only solutions are $a = \pm 1$ with $b = 0$. So only units are ± 1. (b) Lectures. (c) $N(2) = 4$. If $2 = xy$ then $4 = N(2) = N(x)N(y)$. As we don't want either of x or y to be a unit must have $N(x) = N(y) = 2$. But $a^2 + 3b^2 = 2$ is not possible to solve for a and b integers. So 2 is irreducible. (d) We have $(1 + \sqrt{-3})(1 - \sqrt{-3}) = 4$ is divisible by 2. However 2 does not divide either factor as the result would be $1/2 \pm 1/2\sqrt{-3} \notin \mathbb{Z}(\sqrt{-3})$.

Professor Michael Murray
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