

Groups and Rings III 2010

Tutorial Exercise 5.

Please try before the tutorial on Tuesday 18th May.

1. Consider the group S_3 . Deduce everything you can about its Sylow subgroups using the Sylow theorems. Check the answers explicitly by identifying all the Sylow subgroups and either showing they are conjugate to each other or normal as required by the theorems.
2. Find all groups of order 91. Don't just quote the result from class but explain in full.
3. (a) Show that no group of order 40 is simple.
(b) Is there a finite group with 12 Sylow 3-subgroups? Give reasons for your answer.
4. Show that G is a p -group (i.e has order a power of the prime p) if and only if every element of G has order a power of p . (Hint: Cauchy's theorem)
5. Show that no group of order 1000 is simple.
6. Find all groups of order 133. Don't just quote the result from class but explain in full.