Groups and Rings III 2010

Class Exercise 1.

Please hand up solutions in the lecture on Thursday 12th March.

1. Consider the group

$$GL(2,\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \; a,b,c,d \in \mathbb{R} \right\}$$

of 2×2 invertible matrices with real entries with binary operation being matrix multiplication. For each of the following prove if they are, or are not, subgroups of G. If they are subgroups show if they are or are not abelian.

(a) The set D of all diagonal matrices, i.e. $b = c = 0$ in the definition.

(b) The set B of all upper triangular matrices, i.e. $c = 0$ in the definition.

(c) The set H of all matrices whose determinant is π.

2. Consider the group

$$U_{18} = \{z \in \mathbb{C}^\times \mid z^{18} = 1\}$$

with generator $\omega = \exp(i\pi/9)$.

(a) What are the orders of ω^9 and ω^7?

(b) Find all subgroups of U_{18} and draw the subgroup lattice.

3. Recall that in class we defined the quaternion group as $\mathbb{H} = \{\pm 1, \pm i, \pm j, \pm k\}$ with multiplication defined by letting the identity be 1, assuming that -1 commutes with everything else and that also

$$ij = -ji = k, jk = -kj = i, ki = -ik = j, i^2 = j^2 = k^2 = -1 \quad \text{and} \quad ijk = -1.$$

(a) Calculate the elements $ik^2(1)ji$ and $i^3j^2(-1)k$.

(b) Find a subgroup of $\mathbb{H}$ of order 2.

(c) Find a subgroup of $\mathbb{H}$ of order 4.

(d) Find two elements of $\mathbb{H}$ that generate $\mathbb{H}$. (Prove that they generate $\mathbb{H}$).