School of Mathematical Sciences
PURE MTH 3007
Groups and Rings III, Semester 1, 2009

Week 12 Summary

Week 12 — Lecture 25 — Tuesday 2nd June.

Lemma 13.10. *Let D be a PID and let $a_1, a_2, a_3, \ldots$ be a sequence of elements of D such that for each i, $a_{i+1}|a_i$. Then for some N, a_n is an associate of a_N for all $n > N$.*

13.4. Polynomial rings as UFDs.

Theorem 13.11. *If D is a UFD then so is $D[x]$.*

Corollary 13.12. *If D is a UFD so also is $D[x_1, \ldots, x_n]$.*

Hence, in particular, $\mathbb{Z}[x], F[x,y], F[x,y,z]$ are UFDs.

13.5. Relationships between classes of rings.

$$ED \subset PID \subset UFD \subset ID \subset \text{ commutative rings with 1.}$$

Examples:

EDs	$\mathbb{Z}, F[x], \mathbb{Z}(i), \mathbb{Z}(\sqrt{2})$
PIDs which are not EDs	$\{\frac{m}{2} + \frac{n}{2}\sqrt{-19} \mid m, n \in \mathbb{Z}\}$
UFDs which are not PIDs	$\mathbb{Z}[x], \mathbb{Z}[x,y], F[x,y]$
IDs which are not UFDs	$\mathbb{Z}(\sqrt{-5}), \mathbb{Z}(\sqrt{10})$
Commutative rings with 1 which are not IDs	$\mathbb{Z}_m$, m composite.