Algebraic Topology IV 2008

Assignment 2 – Due Thursday 25th September

1. Consider the tangent bundle to the circle TS^1 which is defined as

$$TS^1 = \{(u,v) \mid \|u\|^2 = 1, \quad \langle u,v\rangle = 0\} \subset \mathbb{R}^2 \times \mathbb{R}^2.$$

Show that the inclusion $\iota\colon S^1 \to TS^1$ defined by $\iota(u) = (u,0)$ is a homotopy equivalence and calculate the fundamental group of TS^1. (You may assume known the fundamental group of the circle S^1.)

2. Prove that S^n is path connected for $n > 0$.

3. Is $S^1 \times S^2$ homotopy equivalent to S^3 ? Give reasons.

4. Calculate $\pi_1(S^1 \times S^5 \times \mathbb{R}^{437})$.

5. Let $m < n$ and define an m-dimensional plane $W_m \subset \mathbb{R}^n$ by

$$W_m = \{(x^1,\dots,x^m,0,0,\dots,0)|(x^1,\dots,x^m) \in \mathbb{R}^m\}.$$

Calculate the fundamental group of $\mathbb{R}^n - W_m = \{x \in \mathbb{R}^n \mid x \notin W\}$. State clearly any results that you use.

6. Define real projective space, $\mathbb{R}P_k$, to be the set of all lines through the origin in $\mathbb{R}^{k+1}$. Define a map $p\colon S^k \to \mathbb{R}P_k$ from the sphere to real projective space by mapping a unit vector to the line containing it. Assuming that p is a covering map calculate $\pi_1(\mathbb{R}P_k)$.