Honours/MPhil Courses 2015

SCHOOL OF MATHEMATICAL SCIENCES
THE UNIVERSITY of ADELAIDE

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Course Descriptions:

Research Methodologies (Semester 1 only)

This course covers a number of practical research issues including:

(a) Project and seminar assessment: criteria, components and aims
(b) Code of conduct for research
(c) Intellectual property, copyright and ethics
(d) Writing scientific English: mathematics, sentences, paragraphs and evidence.
(e) Library and web sources, literature reviews.
(f) Materials and data management, computer tools
(g) Professional presentations
(h) LaTeX: separate logical composition from physical layout
(i) Research project management: planning stages, communication and records
(j) Writing grant applications
(k) Providing evidence

Applied Topic A
Advanced Stochastic Processes

Randomness is an important factor in modelling and analyzing various real-life situations. This course covers some key topics in continuous-time stochastic processes: measure-theoretic probability, filtration, martingales, Brownian motions and reflected Brownian motions, Ito integrals, convergence of processes, functional limit theorems, and applications to insurance, environmental modelling, and finance.

Prerequisites: Students should have some background in probability and stochastic processes (for example, discrete-time or continuous-time Markov chains).

Applied Topic B
Modelling and Simulation of Turbulent Flows

Turbulent fluid flows are important in many problems of technological and scientific interest, including vehicle drag reduction, energy production and climate prediction, to name a few. The dynamics of turbulence are governed by the Navier-Stokes equations, which are a system of nonlinear partial differential equations, for which no general solution has yet been found. Approximate solutions of the Navier-Stokes equations can be found numerically, but turbulent flows exhibit such a huge range of spatial and temporal scales that such computations are often infeasible, even on the biggest supercomputers. Consequently, simplified mathematical models that account for the effects of turbulence are necessary in order to obtain predictions of turbulent flows.

This course will cover the mathematical description of turbulent flows, the numerical methods needed to solve the Navier-Stokes equations, and some of the models used to predict turbulent flows. Topics include: Navier-Stokes equations, boundary layers, flow stability, spectral and finite difference solutions of the Navier-Stokes equations, mathematical description of turbulence, Reynolds-Averaged Navier-Stokes Simulations (RANS), Large-Eddy Simulations (LES), and the stretched-vortex model. The course will have significant computational content.

Assumed knowledge includes Fluid Mechanics III, Modelling with ODEs III and PDEs and Waves III.

Applied Topic C/APP MATH 4050
Modelling and Simulation of Stochastic Systems

The course provides students with the skills to analyse and design systems using modelling and simulation techniques. Case studies will be undertaken involving hands-on use of computer simulation. The application of simulation in areas such as manufacturing, telecommunications and transport will be investigated. At the end of this course, students will be capable of identifying practical situations where simulation modelling can be helpful, reporting to management on how they would undertake such a project, collecting relevant data, building and validating a model, analysing the output and reporting their findings to management. Students complete a project in groups of two or three, write a concise summary of what they have done and report their findings to the class. The project report at the end of this course should be a substantial document that is a record of a student's practical ability in simulation modelling, which can also become part of a portfolio or CV.

Topics covered are: Introduction to simulation, hand simulation and computer simulation, review of basic probability theory, introduction to random number generation, generation of random variates, analysis of simulation output, variance reduction techniques and basic analytic queueing models.

Pure Topic A
Algebraic Topology

The aim of Algebraic Topology is to classify topological spaces up to continuous deformations by associating algebraic invariants such as numbers, or groups, to each space. Algebraic objects are assigned in such a way that ``natural" operations on the latter correspond to "natural" operations on the former---continuous maps might correspond to group homomorphisms, homeomorphisms to isomorphisms, etc. In this way, it is often possible to distinguish between different topological spaces by demonstrating that certain associated algebraic objects are not isomorphic.
The core of the subject began with Poincaré in the 1890s and crystallised with the work of Eilenberg and MacLane in the early 1950s. The algebraic topology of today is a very broad and highly generalised area that has pervaded much of contemporary mathematics.

This course serves as a thorough introduction to fundamental concepts in algebraic topology. The goal is to develop some intuition for how algebraic topology relates to concrete topological problems and how to apply algebraic methods to solve such problems. Topics to be covered include the fundamental group, covering spaces, Brouwer fixed point theorem, homology theory, cohomology theory and additional topics if time permits. Although algebraic topology primarily uses algebra to study topological problems, using topology to solve algebraic problems is sometimes also possible. This will be demonstrated by providing a new proof of the fundamental theorem of algebra.

Pure Topic B
Differential Geometry

The central objects of study in differential geometry are manifolds. These are objects that “look like” Euclidean space up close, but can be more complicated when viewed from a distance. The standard examples are surfaces in space: spheres, ellipsoids, tori, and such like. These are two-dimensional manifolds because they look like R2 locally. Einstein's general theory of relativity describes the universe in terms of a four-dimensional manifold equipped with some extra structure.

Manifolds are of fundamental importance in both mathematics and physics, as well as many other disciplines, and they are often studied using techniques of advanced calculus: this is the “differential” in the title. The aim of this course is to provide students with the basic concepts and results associated with manifolds. These will include vector fields, differential forms, Stokes' theorem (the generalisation of the fundamental theorem of calculus to manifolds), de Rham and Cech cohomology, vector bundles, connections and characteristic classes.

A proof of the celebrated Gauss-Bonnet theorem will be given towards the end of the course, but this will differ considerably from the proof given in the Level III course Geometry of Surfaces. The emphasis in that course was more on the geometry as opposed to the differential in this course.

Assumed knowledge for Differential Geometry is multivariable calculus of some form. Other courses that would be useful background, but will not be assumed, are Algebra II and Topology and Analysis III.

If there is sufficient time and interest, some additional non-examinable lectures on gauge theory and the topology of four-dimensional manifolds will be given at the end of the course.

Stats Topic A
Advanced Statistical Inference

This course will introduce students to the theory and practice of modern statistics. There will be five chapters covering the following topics. **1.** *Statistical inference:* cumulants, the cumulant generating function, natural exponential family models, minimal sufficient statistics and completeness. Ancillary statistics, the profile log likelihood, marginal and conditional inference. **2.** *Model choice:* principles of model selection, cross-validation (CV), Akaike's Information Criterion (AIC), Network Information

Criterion (NIC), and the Kullback-Leibler distance. **3.** *Bootstrap methods of inference:* non-parametric bootstrap estimates of an unknown sampling distribution, the bootstrap for assessing statistical accuracy, and bootstrap confidence intervals. **4**. *Survival analysis:* censoring and truncation, failure time distributions, likelihood inference for parametric models, the non-parametric Kaplan-Meier estimator, the semi-parametric proportional hazards regression model (Cox model) and partial likelihood. **5**. *Modern regression:* Alternative approaches to regression *Applications:* throughout the course, applications will be made to a range of subject areas using the statistical package R.

Pre-requisites: It will be assumed that students have attained at least a pass in Mathematical Statistics III (STATS 3006) and Statistical Modelling III (STATS 3004), or have equivalent knowledge.

Stats Topic B
Topics in Bayesian Statistics

Bayesian statistics is concerned with "learning from data". To a large extent, this means updating information in the light of experimental evidence. This course, which concentrates on the ideas behind, and implementation of, Bayesian statistical inference, assumes that students are familiar with the material in a mathematical statistics course, such as Math Stats III — such as random variables, common distributions, transformation of variables, testing and estimation, large sample behaviour of estimators and test statistics.

Concepts of probability

Exchangeability

Basic Bayes concepts (priors, posteriors)

Types of prior distributions

Bayesian inference based on the posterior

Predictive distributions

Large sample results

Hierarchical models

Multivariate/multiparameter Bayes models; shrinkage,

Connections between Bayesian estimation and frequentist decision theoretic estimation

Importance sampling, MCMC, Gibbs sampling

Applied Topic D
Dynamical Systems

This course provides an introduction to the theory of continuous and discrete dynamical systems with a particular emphasis on bifurcations and routes to chaotic behaviour. We begin by studying examples of how low-dimensional dynamical systems arise from approximations to complex physical systems. We observe that these systems can exhibit changes in behaviour as governing parameters are varied, including chaotic dynamics (extreme sensitivity to initial conditions). To understand this behaviour we study the stability and bifurcations of periodic structures, bringing together results from linear algebra, multivariable calculus, differential equations, topology and group theory.

Applied Topic E
Matrix Analytic Methods in Stochastic Modelling

Matrix-analytic methods are popular tools in stochastic modelling because they allow the construction and analysis, in a unified and algorithmically tractable manner, of a wide class of stochastic models. The methods have been applied in various areas, including health, finance and most notably performance analysis of communication systems. This course presents the basic mathematical ideas and algorithms that are part of the matrix-analytic methods. The approach uses probabilistic arguments to the fullest extent and demonstrates the unity of the argument in the whole theory. It also reveals the stochastic process at work within the computational procedures.

Applied Topic F

Applied Methods in Biology and Engineering

The goal for this class is for students to develop a fundamental understanding of how mathematics is applied as a tool to aid in studying complex systems in biological and engineering sciences. We will thoroughly investigate case studies in several fields including: Neuron Action Potential Propagation, Cardiac dynamics, Tumor/cancer growth and Pattern formation. Graded work will include a mix of theoretical and computational homeworks, culminating in a final exam.

Practically speaking, this class is designed for advanced undergraduate, honours, MPhil and beginning PhD students in the mathematical, physical, and biological sciences with a solid mathematical background, i.e., Linear Algebra and Differential Equations.

The course pre- requisite (which can be waived with instructor approval) Modelling with Ordinary Differential Equations III (APP MTH 3021) and may be taken simultaneously with this course. Also note that familiarity with MATLAB or other programming language is assumed (prerequisites include classes which use MATLAB). The list of mathematical tools and methods that we will learn in this course include:

1) Matched asymptotics, multi-scale modelling
This method will be taught via Michaelis-Menton kinetics, Fitzhugh-Nagumo and Hodgkin- Huxley models.

2) Solving moving boundary problems, traveling wave solutions
This method will be introduced while solving advection-diffusion-reaction equations for tu- mor growth, pattern formation and the Fisher's equation.

Pure Topic D
Functional Analysis

In general, functional analysis can be defined as the study of infinite dimensional vector spaces and operators on these spaces. Functional analysis as a separate mathematical subject emerged about a century ago, motivated in part by the success of using Hilbert spaces to rigorously justify Fourier series expansions for functions on the unit circle. In the extension of this theory to other settings, fundamental questions in functional analysis appear: (i) What are the appropriate topologies to put on the infinite dimensional vector spaces that appear as spaces of functions? (ii) To what extent do linear differential operators acting on spaces of functions (like $d/d\theta$ acting on functions on the circle) behave like linear transformations on finite dimensional vector spaces?

The first question is highly nontrivial and depends on the context, precisely because infinite dimensional vector spaces have many inequivalent norm topologies. Investigating the second question quickly leads to the realization that even the simplest linear differential operator like $d/d\theta$ is discontinuous in the most reasonable topologies. On the other hand, standard operators like Green's operators (e.g. $(d/d\theta)^{-2}$, roughly speaking the inverse of the Laplacian on the circle) are continuous in these topologies. Therefore, functional analysis naturally splits into the study of continuous operators on function spaces and discontinuous operators -- both are important.

More specifically, this course will cover the basic topologies on infinite dimensional vector spaces, including Hilbert spaces (inner product topologies), Banach spaces (norm topologies), and Frechet spaces (topologies built to handle spaces of smooth functions). Specific examples will include L^p and Sobolev spaces. We will restrict attention to continuous linear operators between these spaces, with applications to the study of differential operators. We will discuss the spectral theorem about diagonalisation of linear operators on Hilbert spaces with Fourier series as the guiding example. As time permits, we'll discuss more advanced results like the Hahn-Banach theorem and the theory of distributions, the rigorous treatment of delta functions.

Prerequisites

Topology and Analysis III, and Integration and Analysis III. A good background in linear algebra and real and complex analysis is desirable, as well as some familiarity with groups and manifolds.

Pure Topic E
Symmetry in Differential Geometry

Many familiar manifolds are "homogeneous," they look the same at each point even when some extra structure is taken into account. A good example is the sphere with its usual "round" metric. To make this precise, the notion of a "Lie group" is useful: its definition combines the concepts of a group and a smooth manifold. Lie groups themselves are homogeneous and are well captured by an infinitesimal and purely algebraic notion known as a "Lie algebra." So this course is about Lie algebras, Lie groups, and their actions on smooth manifolds. The round sphere is homogeneous under the action of its isometries, which is a Lie group that can be described in terms of matrices, as can its Lie algebra (and this will be true for all the Lie groups in this course).

But there are many more homogeneous structures, even on the sphere. For example, the round sphere is also homogeneous under conformal, i.e. angle-preserving, symmetries or projective, i.e. geodesic-preserving, symmetries. Each of these variations comes with its own type of differential geometry. This course will catalogue the various possibilities and explore the associated differential geometries. Of particular interest, especially in physics, are the differential operators that respect these symmetries. Using methods from the theory of Lie algebras (but always expressed in terms of matrices), some classifications of these operators will be obtained.

Key Phrases: Homogeneous space, Homogeneous bundle, Lie group, Lie algebra, Conformal differential geometry, Contact geometry, Parabolic geometry, Invariant differential operator.

Assumed Knowledge: This course naturally follows on from the Differential Geometry honours course in Semester 1. Basic linear algebra and group theory will be useful.

Stats Topic D