THE UNIVERSITY of ADELAIDE

School of Mathematical Sciences

Vacation Scholarship Projects

2016/2017

Project Titles

Spreading wavefronts
A range of quite different phenomena can sometimes be characterised as a wavefront spreading into new regions. Examples include tsunamis, the population density of an invasive species, the cancer cell density in a spreading solid tumour, and the temperature profile in a bushfire. This project will focus on numerical solutions of partial differential equations modelling one or more of these phenomena in order to determine how fast the front is spreading. This project has applicability in for example understanding the factors leading to a rapid spread of cancer, or in evaluating how some intended treatment may mitigate its spread. *Prerequisites: a second-year differential equations course, and strong programming capability in Matlab.*

Characterising chaos
This project will study the concept of chaos from a mathematical viewpoint. Qualities which characterise chaos (e.g., sensitive dependence on initial conditions, presence of periodic orbits of all periods, uncountably many aperiodic orbits, and dense orbits) will be studied. Methods for analytically proving that a system is chaotic (presence of a "horseshoe," intersection of stable and unstable manifolds, Melnikov's method) will be applied to different systems arising in applications. *Prerequisites: a second-year differential equations course and an interest in theoretical methods of differential or difference equations.*

Unsteady flow barriers
In unsteady (time-varying) flow fields, there are usually important invisible flow barriers which move with time. Examples include the boundary of an area ("the forbidden zone") near the coast of Florida into which the Deepwater Horizon oilspill did not leach, and the edge of the Antarctic Circumpolar Vortex ("the ozone hole"). This project will focus on understanding and using recently developing methods for identifying such flow barriers, and (depending on student background) will choose from George Haller's methods of curves of maximal attraction, Gary Froyland's transfer operator methods and/or Jean-Luc Thiffeault's topological complexity of curves method. *Prerequisites: a second-year differential equations and fluid dynamics course, plus either a strong theoretical mathematics or programming background.*

Numerical computation of stable and unstable manifolds
The "flow barriers" described in the previous project can be recast theoretically as "stable and unstable manifolds." In two-dimensional flows, for example, these would be time-varying curves which exhibit complicated intersections patterns with each other, and forms the template which governs how fluids mix. Numerically determining these for time-varying (unsteady) velocity fields remains a challenging problem. In this project, the student will build on a currently existing numerical algorithm to seek highly refined stable and unstable manifold curves. *Prerequisites: strong programming capability in Matlab.*

I am also open to topics in other areas, to be agreed upon after discussion with me.

Supervisor: Dr Sanjeeva Balasuriya

I am happy to discuss possible topics for a research project in areas such as differential geometry, topology and complex analysis.

Supervisor: Dr David Baraglia

I am happy to supervise projects in the areas of finite geometry and combinatorics. Interested students can email me to arrange a meeting and we can discuss possible project ideas.

Supervisor: Dr Susan Barwick

Stochastic Modelling using Structured Markov Chains

In this project we will investigate the mathematical properties of some very recent stochastic models. These models have been developed from the basic principles used in a field known as "Matrix-analytic Methods" (or MAM) where simple exponentially distributed lifetimes are replaced by lifetimes from more complex distributions. When done carefully, the analysis of the whole model becomes matrix-based, rather than scalar-based, hence the name. Of course, this brings all sorts of challenges (for example, the square root operation no longer makes any sense) and requires a much closer connection to the physical model itself. This, and an associated emphasis on computational algorithms, are the main features of this area of stochastic modelling.

Supervisor: Professor Nigel Bean

Wave propagation in random media

Even the most simple wave problems become non-trivial and produce interesting behaviours when randomness is introduced. Students can investigate the effects of different types of randomness and in different settings, using numerical and/or analytical techniques.

Modelling sea ice dynamics & thermodynamics

Climate change is weakening and fragmenting the sea ice that covers vast areas of the Arctic Ocean. An important consequence is that the ice cover is now far more dynamic, and has more potential to melt in the summer and grow in the winter. Students can undertake a project to investigate this phenomenon.

Mathematical and experimental modelling

Combining mathematical and experimental models is a powerful way to accurately model real world phenomena. Students can conduct a project to develop a mathematical model of water waves and conduct simple wave tank experiments to validate the model.

Supervisor: Dr Luke Bennetts

Quantifying and modelling yeast colony spatial patterns

Yeasts colonies can forage for food by either the process of filamentous growth, or the formation of a biofilm. Both are highly non-uniform spatial-temporal processes, often producing complex spatial patterns. The overall goal of this project is to develop models that predict the time evolution of colony morphology. However, an important part of this work is the spatial quantification of yeast growth experiments, which can be used to validate modeling predictions. Therefore, one of our aims is to develop user-friendly open source software that can process experimental images and provide metrics on the spatial patterning of colony morphology. The second aim is the modeling itself, and both continuum and discrete approaches could potentially be explored during the course of the project. The data for the statistical analysis and model validation will be obtained from laboratory experiments.

Supervisor: Dr Ben Binder

Evolution of life cycles in multicellular collectives

Simple multicellular collectives, such as chains of bacteria, are of interest because they are the starting point for all further multicellular evolution. The emergence of life cycles and basic developmental processes are key steps in multicellular collectives becoming fully-fledged organisms. This project will involve using stochastic techniques to build models of these biological systems. These will then be studied using a combination of analytic and simulation methods.

Inference methods for epidemic models

A crucial part of epidemic modelling is to characterise a disease in the early stages of an outbreak so as to inform public health policy and implement measures to slow its spread.
This project will involve learning methods for computational Bayesian analysis and applying them to household models of influenza.

Supervisor: Dr Andrew Black

All things considered, what is the optimal direction in which to point a solar panel?

This project could be tackled by a student who has done Maths I. The degree of difficulty is really up to the person undertaking the project, as there are many issues that could be taken into consideration---for example, the efficiency of solar panels depending on their temperature; the amount of dust in the atmosphere depending on the time of year; etc.
With very basic assumptions, the problem should not be too hard.

How would we calculate the intrinsic distance between two points on a surface?

This project could be tackled by a student who has done Multivariable & Complex Calculus and Real Analysis. There are various issues to be considered, such as how we define the length of a curve; how we might find a curve that minimizes arc length (geodesics); how we might solve the differential equations for a geodesic.

The Dirichlet problem.

The classical Dirichlet problem is that of finding a function with specified boundary values whose Laplacian is some given function.
Proving the existence of such a solution is technically quite difficult, but it serves as a prototype for solving many problems in analysis. This project would be appropriate for someone who has done the course Integration & Analysis III

Supervisor: Dr Nicholas Buchdahl

Nanoscaled oscillating systems

Nonoscaled structures such as carbon nanotubes and fullerenes undergo interactions described by van der Waals forces. At very small scales these interactions can lead to extreme accelerations, velocities and, in the case of oscillating systems, frequencies. By modelling the structures as surfaces with uniform atomic densities and the van der Waals interactions using a 6-12 Lennard-Jones potential, we can make predictions regarding these systems including deriving a formula for the frequency which is in good agreement with molecular dynamics simulations. In this project the student will look at models to calculate the force and predict the behaviour of various oscillating systems.

Geometries and geometric issues of nanostructures

It is clear from the various structures seen at the nanoscale that the complex interactions of these structures often lead to symmetric conformations. So in satisfying a minimum energy constraint the system often adopts a symmetric structure that shares the energetic costs of bending and stretching covalent bonds equally to all components in the structure. By assuming a symmetric conformation up front, it is possible to reduce fundamentally complex problems of molecular structure to problems with are more mathematically tractable and thereby derive results which can be confirmed by experiment and simulation and can also be used to predict ideal systems and novel structures in certain extreme cases. In this project the student will study models for nanostructures such as nanotubes, cones and spheres (buckyballs) with the aim to provide more precise predictions of structural parameters like length and radius.

Supervisor: Dr Barry Cox

Interactions between cells in tissues and individuals in groups

The collective behaviour of animals in swarms, or cells in tissues, is governed by the interactions between the individuals in the group. We can use mathematical models to understand how different types of inter-individual interaction lead to different arrangements of cells in tissues, or movements of swarms. These models can be computational agent-based models, in which individual cells or organisms are represented, or systems of partial differential equations for the densities of each species, which can be investigated using a combination of analytical and numerical methods. The student is welcome to choose whichever approach best suits their interests.

Supervisor: Dr Edward Green

Noncommutative topology

All properties of nice enough topological spaces, specifically locally compact Hausdorff spaces, are encoded in the continuous functions on them. These continuous functions form a vector space, and also a commutative ring, via pointwise multiplication. These structures together make this function space an algebra, and it actually has the further structure of a C*-algebra. So one can study a locally compact Hausdorff space via its algebra of continuous functions, which is a viewpoint that sheds new light on many questions and constructions.

But more generally, one can also study noncommutative C*-algebras, which are interpreted as algebras of functions on “noncommutative spaces”. This can be used to study difficult spaces, such as non-Hausdorff ones. In this project, we will look at what C*-algebras are, and how tools from topology can be generalised to this framework.

Prerequisites: Groups and Rings, Topology and Analysis.

The geometry of classical mechanics

The mathematical language of classical mechanics is symplectic geometry or Poisson geometry. These areas of geometry are about spaces with structures that allows one to state Newton's laws on them. These spaces should be though of as the sets of all positions and momenta a mechanical system can have.

E.g. for a double pendulum (a pendulum hanging from another pendulum), the space is the 2-dimensional torus plus all of its tangent planes. The torus, i.e. the Cartesian product of two circles, parametrises the position angles of the two pendulums, while the tangent planes parametrise the momenta, or velocities, of the two pendulums.

In this project, we will see how to formulate the laws of classical mechanics in terms of Poisson or symplectic geometry. We will explore some aspects of this point of view and/or apply the theory to a concrete example.

Groups and representations

Representations of a group are ways in which this group encodes the symmetry of a physical, chemical or mathematical object. We can look at various aspects, examples and applications of representation theory, especially for Lie groups. These are essentially groups of matrices.

I am also happy to discuss other projects in differential geometry, Lie group theory or analysis on manifolds. Any project can be adapted to the student's knowledge.

Supervisor: Dr Peter Hochs

The Classical Groups

The classical matrix groups are defined as invariance groups of certain multilinear maps on real, complex and quaternionic vector spaces. The aim is to study them as curved spaces within the vector space of n-by-n matrices and to establish some of the interesting relations between them that exists for small n. Further exploration can be related to topological properties of the classical groups or to the quotient spaces arising from them.

Quaternions and Octonions

In a similar way how the complex numbers are constructed from the reals, the quaternions are constructed from the complex numbers and the octonions from the quaternions. Thus, both can be considered as generalisations of complex numbers to higher dimensions. Many interesting algebraic and geometric phenomena are related to the quaternions and octonions. To explore these features and relations is the aim of the project.

Other possible projects

Apart from these two topics I am happy to discuss any topic that is related to differential geometry.

Supervisor: Dr Thomas Leistner

#selfies and emotional expressiveness

Bizarrely, there is evidence a facial asymmetry in the way human beings express emotions in photographs: people who face the camera with their right side leading are considered to be more emotionally expressive than those who lead with their left side. This effect has recently been studied in photographs of various groups of people, including academics and doctors. In this project we will collect and analyse selfies from Instagram to see how this facial asymmetry effect relates to different personality types on social media. After collecting a dataset and categorising individuals into left-facing and right-facing groups we will then test for differences in language using a combination of statistical and sentiment analysis techniques.

This project will involve some data mining and analysis using tools like Python and/or MATLAB – familiarity with one or both will be very desirable (and you'll learn more useful tools such as how to access data using APIs over the course of the project!).

Do the rich get richer on reddit?

The "Matthew Effect" is the widely-observed phenomenon whereby the rich get richer, or popular get more popular. Very famous models such as "preferential attachment" in network science have been used to explain diverse phenomena such as word distributions in language, scientific citation networks and more. In this project we will investigate whether the Matthew Effect exists in online social media, particularly in submissions to the popular website reddit. We'll collect time-resolved data on submissions to various subreddits using the reddit API, and then analyse the statistics of these data to try and detect the Matthew Effect. Along the way we'll encounter Simon's model, preferential attachment, and other interesting topics in complex systems science.

This project will involve some data mining and analysis using tools like Python and/or MATLAB – familiarity with one or both will be very desirable (and you'll learn much more as the project goes on!).

Studying humans 'in the wild' via social networks and Big Data

With the explosion in recent times of data from large-scale social networks such as Facebook and Twitter comes unprecedented opportunities to bring quantitative and computational methods to bear on problems in social, cultural and political science. This emerging field of computational social science blends mathematical and statistical techniques with computer science and very large data sets to study and predict the behaviours of groups of people based on their online activities — by observing them 'in the wild'.

We will analyse unique datasets comprising millions of tweets to look for trends in human dynamics. There are many potential topics here, possible case studies include:

1. Senitment analysis of the #qanda audience for the purpose of real-time political polling;
2. Extracting daily mobility patterns from geolocated tweets to infer human dynamical patterns – these have particular relevance in fields like disease modeling;
3. Can you predict the #hottest100 using social media data? In particular, how close can you get using Triple J's playlist data coupled with mentions on social media? (This will probably require some scrounging for data…)

While rather computational in nature (we will be making use of programming languages like Python or MATLAB to make sense of these data sets; familiarity with one or both of these would be desirable), this project will provide the opportunity to engage with large, real-world data sets, and to do original research in an exciting new field.

Predictability, information theory and culturomics

'Culturomics' is an emerging form of computational lexicology that studies human behaviour and cultural trends through digital texts. Recent high-profile studies have analysed Google Books, broadcast media and digitised news articles to find large-scale cultural patterns. We will use techniques borrowed from information theory and the emerging field of sentiment analysis to study culturomical questions in a new way: Is literature becoming more predictable? Are movies becoming more generic and 'dumbed-down'? We will apply new quantitative techniques to extremely large text-based datasets comprising tens of thousands of books from Project Gutenberg and over 1000 film scripts in order to study these questions.

We will make extensive use of programming languages such as Python and/or MATLAB in this project; familiarity with one or both of these would be desirable.

Supervisor: Dr Lewis Mitchell

Differential geometry
Mathematical physics

I am happy to discuss possible topics in the areas of differential geometry and mathematical physics. Interested students should email me and arrange a meeting**.**

Supervisor: Professor Michael Murray

Explode or Extinct?

Branching processes are mathematical models used to describe and analyse how populations evolve over time, with applications in many areas such as biology, epidemiology, computer science and image processing. Students can study the effects of initial starting points and growth rates on the eventual size of the population, via algorithms and/or probabilistic analysis.

What does the Apollo spacecraft and Wall Street have in common?

Diffusion processes played an important role in estimating the trajectories of the Apollo spacecraft on its way to the Moon and back, as well as in building the myriad of intricate models that is today's financial world. Students can study the effects on key properties of diffusion processes when we impose boundary constraints on these models.

How likely is that?

Natural disasters, financial crises, large-scale accidents and system breakdowns are examples of phenomena with extreme consequences that occur with non-negligible frequencies. Heavy-tailed models are able to capture adequately the behaviour of this type of phenomena but are often intractable. Students can study a class of models associated with infinite-phase-type distributions that can replace heavy-tailed models, from numerical and/or theoretical perspectives.

Supervisor: Dr Giang Nguyen

Help develop a toolbox for multiscale simulation and analysis

Often a researcher/practitioner has a detailed and trustworthy computational simulation of some problem of interest. The simulation is written in terms of micro-field variable values. Typically a desired simulation over large space-times would take far, far too long. We are developing and proving techniques to enable such simulation using projective integration and patch dynamics. The project over summer is to help create a Matlab toolbox for users around the world to automatically use the techniques on problems of interest.

Establishing critically useful theory for modelling.

Recently we are understanding more and more about the fundamental relationships between mathematical models at different levels of detail. It turns out that the so-called Centre Manifold Theory provides the rigorous support needed for such understanding. However, the extant theory only deals with the mathematical ideal case of exact results for precise systems on infinite times in an unknown domain. In practice we need theory valid for uncertainly known systems over finite times in a known finite domain. The project is to continue work developing theory to generate such useful practical theorems to underpin mathematical modelling.

Supervisor: Prof Tony Roberts

Modelling variation in Australia's climate change data.

The Bureau of Meteorology maintains several important datasets as part of its climate science programme. One of these is the Australian Climate Observations Reference Network - Surface Air Temperature dataset (ACORN-SAT) of daily temperature records dating back 100 years. The purpose of ACORN-SAT is to monitor climate variability and change in Australia. There are however numerous sources of temporal and spatial variability which impact on the accuracy and precision of the daily temperature measurements in ways that are not yet understood or quantified. This summer research project will begin looking at this problem by investigating the sources of variability then using hierarchical statistical modelling to describe the observed uncertainty and its impact on the temperature record. The analysis will be conducted in R and will use K-fold cross validation of training and test datasets to validate the statistical models. This research project is intended for a student planning to go on to study for an Honours or Masters degree.

Longitudinal data analysis for improving patient outcomes.

Longitudinal data arise when repeated measurements are made on patients over time and the analysis of these data represents an important area of medical statistics. In this summer research project you will analyse data on patient outcomes from knee and hip replacement surgery conducted at the Royal Adelaide Hospital. The aim of the study is to develop a model which predicts post-operative patient pain and function using individual (pre-operative) patient multi-morbidity data. The ultimate purpose of the study is to provide clinicians with a predictive tool for assessing the potential benefits of knee and hip replacement surgery. The project will involve regression modelling in R using cross-validation to select the best predictive model. This project will form the basis of further research to develop a diagnostic tool for surgeons.

Sample size calculations for classification in omics studies.

An important part of study design is estimating the required size of the study to be conducted to reasonably answer the scientific question(s) being asked. Having the appropriate number of observations (sample size) ensures there is sufficient statistical power to detect the real biological effects of interest. Sample size calculations for microarray and proteomics studies present special challenges however, owing to the special features of the technologies and the high-dimensional nature of the data. To date, most sample size calculations have concentrated on detecting the significance of individual gene expression levels or adjusting for multiple comparisons. This research project will develop sample size calculations for discriminating between classes of observations, in particular, cancer versus non-cancer groups, based on tens to thousands of predictor variables. The techniques used will employ simulation studies in R and the analysis of proteomic gastric cancer data, and statistical power will be assessed by the proportion studies reaching pre-specified levels of correctly classified observations. Omics datasets also suffer from missing data and the techniques to be developed will also accommodate missingness in the data.

Supervisors: Professor Patty Solomon and Dr Tyman Stanford

Algebraic topology or Category theory

I am happy to supervise projects in algebraic topology or category theory. If you are interested, please email me to arrange a meeting to discuss possible topics.

Supervisor: Dr Danny Stevenson

Diffusive transport of chemical signals

An unfertilized mammalian egg is surrounded by cumulus cells. On fertilization of the egg the cumulus cells move away from the egg and this is seen in high speed video as a travelling wave. It is thought that the cells respond to one or more chemical signals from locations on the surface of the egg. This project will explore travelling wave solutions of reaction-diffusion equations with the aim of developing a model to explain the behaviour of the surrounding cells after fertilization of an egg.

Supervisor: Dr Yvonne Stokes

Geometry and topology

I have happy to supervise projects in differential geometry, topology and related areas. If you are interested, please email me to arrange a meeting and we can discuss possible topics.

Supervisor: Dr Raymond Vozzo

Geometry and symmetry

I am happy to supervise summer research topics related to topology and geometry, geometry of noncommutative spaces, Fourier and harmonic analysis, groups and number theory. Interested students are encouraged to email or talk to me to figure out a detailed plan. I have successfully supervised a summer project 2014-15 supported by AMSI Vacation Research Scholarship.

Supervisor: Dr Hang Wang