The Makerbot: Desktop 3D printing

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Star Trek Predictions

- Communicators => Mobile phones
- Personal Access Display Device => iPad
- Transparent aluminium => aluminium oxynitride
- Replicators => ???

http://www.youtube.com/watch?v=pzqW0YaN2ho
CNC

- CNC = Computer Numerical Control
- CNC machines have been around for a while
  - 3D printers are one of many (e.g. Milling machines)
- Recently
  - Drop in price (as low as $800 for a kit)
  - Improvement in how easy they are to build and run
- Various tech
  - Resin, lasers, etc.
  - Makerbot deposits layers of molten plastic
- Lots of maths embedded
  - Control (temperature, position, ...)
  - Representation of objects
    - CSG = Constructive Solid Geometry
  - Algorithms for converting objects to tool-head path
    - Approximation of curves into straight lines

- Like much modern tech, the math is hidden
CSG

Union

Intersection

Difference

http://en.wikipedia.org/wiki/Constructive_solid_geometry
Tool-head path

- Slide the object into horizontal layers
  - Compute intersections of plane with 3D object
- Determine how to fill in the physical region
  - Hollow interior
  - Strong boundary layer
- Approximate curves with straight line segments
- Most efficient to do all at once
  - e.g., compute intersections of triangles in 2D
A simple example: intersection

- Take two regions, and find their intersection

- Approach
  - Triangulation
  - Intersect the triangles
Triangulation
Intersection of triangles

- Quite a few cases

- Plus all the special cases, e.g.,
  - Shared vertex, or vertex on an edge
Triangle intersection

- Important things to check
  - Line intersections
  - Is a point inside, or outside a triangle?
Point-triangle test

- Test whether a point is inside a triangle by checking its on the correct side of each boundary line

- We can do that with triangle areas
Many ways to compute the area of a triangle

Here is one using the determinant of a matrix:

\[
\text{Area} = \begin{vmatrix}
 x_A & x_B & x_C \\
 y_A & y_B & y_C \\
 1 & 1 & 1
\end{vmatrix}
\]

Much quicker than using trig

Order of vertices determines the sign
Area of triangle

- Area measurement has plus or minus sign depending on the order of the points
  - swapping two columns in a determinant changes its sign

\[
\text{Area} = \begin{vmatrix}
  x_A & x_B & x_C \\
  y_A & y_B & y_C \\
  1 & 1 & 1 \\
\end{vmatrix}
\]
Area of triangle

- Area measurement has plus or minus sign depending on the order of the points
  - Changing order of vertices changes the sign of our "Area"

\[
\text{Area}(A, B, C) = -\text{Area}(A, C, B)
\]
Point-triangle test

Compare the sign of $\text{Area}(A, B, C)$ and $\text{Area}(A, D, C)$

We learn if $D$ is on the same side of $AC$ as $B$
Point-triangle test

Compare the sign of $\text{Area}(A,B,C)$ and $\text{Area}(A,D,C)$

We learn if $D$ is on the same side of $\overline{AC}$ as $B$
Obviously...

- Obviously this is simplified
  - I need other bits to do intersection
  - I need many other algorithms
- The point is
  - Lots of maths hidden in something like 3D printing
  - Geometry and Linear algebra ++
- Same math used in other areas
  - CGI used in movies
  - Computer game graphics