List of acronyms

A/D Analogue to Digital Convertor
AR Auto-Regressive
ARMA Auto-Regressive Moving Average
ARIMA Auto-Regressive Integrated Moving Average
CD Compact Disc
CFT Continuous Fourier Transform
CGI Computer Generated Imagery
CWT Continuous Wavelet Transform
DAC Digital to Analogue Convertor
DCT Discrete Cosine Transform
DFT Discrete Fourier Transform
DWF Discrete Wavelet Filters
DWT Discrete Wavelet Transform
EWMA Exponentially Weighted Moving Average
fARIMA fractional Auto-Regressive Integrated Moving Average
fBM fractional Brownian Motion
FFT Fast Fourier Transform
fGN fractional Gaussian Noise
FIR Finite Impulse Reponse
FS Fourier Series
FT Fourier Transform
IFFT Inverse Fast Fourier Transform
IFT Inverse Fourier Transform
IIR Infinite Impulse Reponse
JPEG Joint Photographics Experts Group
LP Long Play (record)
LTI Linear Time-Invariant (filter/system)
MRA Multi-Resolution Analysis (or Approximation)
MA Moving Average
RMS Root Mean Square
STFT Short Time Fourier Transform
WFT Windowed Fourier Transform

Some terminology

Some frequently used terminology (see notes for details)

- **dB:** Decibels (defined WRT a reference power level p_{ref}) by $dB = 10 \log_{10} \frac{p}{p_{\text{ref}}}$. Note that $p = m^2$, so we may write $dB = 20 \log_{10} \frac{m}{m_{\text{ref}}}$
- the **Power Spectrum** of a signal $f(t)$ is $|F(s)|^2$, where $F(s)$ is the Fourier transform.
- **Nyquist:** Assume the spectrum of the signal is zero above a critical frequency f_c. For sampling frequencies $f_s > 2f_c$, aliasing dowsn't occur. If $f_s < 2f_c$ aliasing may become a problem.
- **Dynamic range:** expresses the range of values we can represent in our digital format. Representation with b bits (ignoring the sign bit), the dynamic range is roughly 6dB per bit.
- **Hz** Unit of frequency. 1 Hz = 1 cycle per second
- linear systems and filter terminology: lecture 5-6
  - linear, time-invariant, invertible, memory, causal, stability
  - IIR, FIR
  - high-pass, low-pass, stop-band,
  - pass-band, stop-band attentuation, Gibb's phenomena (see Figure Lecture 05).
  - block diagram, tap
- **white noise:** a random process with a flat power-spectrum.
- **spectral density:** expected power-spectrum of a random process, equal to the Fourier transform of autocovariance.

Some math notation

$\langle f, g\rangle$ is the inner product of f and g. This can be defined in various ways, but unless otherwise specified we use the definition

$$\langle f, g\rangle = \left[\int_{-\infty}^{\infty} f(t) g^*(t)\, dt\right]^{1/2}$$

The norm $\|f\| = \langle f, f\rangle$ can be likewise redefined, but we shall typically use the L^2 norm based on the inner product above.

$L^p(\mathbb{R})$ = the set of functions of the real line $f : \mathbb{R} \to \mathbb{R}$ for which the L^p norm exists, and is finite, i.e.

$$\|f\|_p = \left[\int_{-\infty}^{\infty} |f(t)|^p\, dt\right]^{1/p} < \infty$$

C^n = the set of functions with n continuous derivatives

Functions with **compact support** are zero outside a compact set (e.g. an interval $[-a, b] \in \mathbb{R}$). The **support** of a function is the set closure of the set where the function takes non-zero values.

Common definitions

Complex numbers: $x = a + ib,$ where $i = \sqrt{-1}$

- real part of x is $\Re(x) = a$
- imaginary part of x is $\Im(x) = b$
- complex conjugate $x^* = a - ib$
- Hermitian of a complex matrix $A = [a_{ij}]$ is $A^H = [a^*_{ji}]$.
- identities
  - $e^{ix} = \cos(x) + i\sin(x)$
  - $\cos(x) = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$
  - $\sin(x) = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$

Simple signals

- unit step: $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$
- rectangular pulse: $r(t) = u(t + 1/2) - u(t - 1/2)$.
- sign (signum) function: sgn$(t) = \begin{cases} -1, & t < 0 \\ 1, & t > 0 \end{cases}$
- Delta "function" $\delta(t)$ definition

$$\begin{aligned} \delta(-t) &= \delta(t) \\ \int_{-\infty}^{t} \delta(s)\, ds &= u(t) \\ \int_{-\infty}^{\infty} f(t)\delta(t - t_0)\, dt &= f(t_0) \end{aligned}$$

Signal characteristics

- even: $x(-t) = x(t)$
- odd: $x(-t) = -x(t)$
- any signal $x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t)$ where

  $x_{\text{even}}(t) = \dfrac{1}{2}\left[x(t) + x(-t)\right]$ and $x_{\text{odd}}(t) = \dfrac{1}{2}\left[x(t) - x(-t)\right]$
- Hermitian: $x(-t) = x^*(t)$
- periodic: $x(t+nT) = x(t)$ for any $n = 1, 2, \ldots$, and some $T > 0$.

  The minimal value $T = T_0 > 0$ for which periodic signal $x(t + nT) = x(t)$ for any $n = 1, 2, \ldots$, and some $T > 0$ is called the fundamental period, and has units of seconds.

$$\begin{aligned} T &= \text{ period measured in seconds} \\ f &= 1/T = \text{ frequency measured in Hz} \\ \omega &= 2\pi f \text{ measured in radians per second} \end{aligned}$$

Simple transformations

- time reversal $y(t) = x(-t)$
- time scaling $y(t) = x(at)$
- time shift $y(t) = x(t - t_0)$
- amplitude scaling $y(t) = Ax(t)$
- amplitude shift $y(t) = B + x(t)$.
- for complex signals $x(t) = a(t) + ib(t)$
  - real part $\Re(x(t)) = a(t)$
  - imaginary part $\Im(x(t)) = b(t)$
  - conjugate $x^*(t) = a(t) - ib(t)$
  - magnitude $|x(t)| = \sqrt{a(t)^2 + b(t)^2}$
  - phase angle $\theta(t) = \arctan(b(t)/a(t))$
  - $x(t) = |x(t)|e^{i\theta(t)}$

Fourier series: We can write a periodic function as an (infinite) discrete sum of trigonometric terms, e.g. period 2π

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, \cos(nx)\, dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, \sin(nx)\, dx \end{aligned}$$

Fourier transform: $F(s) = \int_{-\infty}^{\infty} f(t)e^{-i2\pi st}\, dt$

Inverse Fourier transform: $f(t) = \int_{-\infty}^{\infty} F(s)e^{i2\pi st}\, ds$

Example FTs

Function	Transform
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-i2\pi t_0 s}$
$r(t)$	sinc(s)
$e^{-\lvert t\rvert}$	$\frac{2}{4\pi^2 s^2+1}$
$e^{-\pi t^2}$	$e^{-\pi s^2}$
1	$\delta(t)$
$e^{i2\pi s_0 t}$	$\delta(s - s_0)$
sinc(t)	$r(s)$

Discrete Fourier Transformation

A signal $x(n)$, with N data points, has DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-i2\pi kn/N},$$

Inverse DFT

$$x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k)e^{i2\pi kn/N},$$