Transform Methods & Signal Processing
Class Exercise 7: before lecture, Monday 2nd November.

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1 [4 marks] We can define a 2D z-transform as follows:

$$G(z,w) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} g(i,j) z^{-i} w^{-j}$$

Given a 2D z-transform of the following form

$$G(z,w) = (z^{-1} - z)(w^{-1} + 2 + w)$$

describe the type of filter that would result.

2 [6 marks] The Hilbert transform of a signal $f(t)$ is defined by

$$f^+(t) = \mathcal{H}(f) = \int_{-\infty}^{\infty} f(\tau) h(t-\tau)\, d\tau = (f * h)(t)$$

where

$$h(t) = \frac{1}{\pi t}.$$

(a) Given that the Fourier transform of h is

$$H(s) = -i\text{sgn}(s),$$

describe the Hilbert transform in terms of Fourier transforms, and hence derive an inverse Hilbert transform.

(b) Hence or otherwise calculate the Hilbert transform of $\cos(t)$.

(c) Assume a real signal can be written in the form

$$f(t) = A\cos(\phi(t))$$

where $\phi(t) = \omega t + x(t)$ and we assume that $x(t)$ varies much more slowly than the carrier ωt, and so can be treated as a constant. Consider the complex signal

$$c(t) = f(t) + i f^+(t)$$

Calculate the magnitude $|c(t)|$ and the phase angle of the signal, and relate these to the original signal.

(d) Show that the inverse Fourier transform of $H(s)$ is $h(t)$, or visa versa (using results given in lectures).