ADELAIDE UNIVERSITY AUSTRALIA

Examination in School of Mathematical Sciences

Semester 2, 2004

MODELLING TELECOMMUNICATIONS TRAFFIC
APP MATH 4014

Official Reading Time:	10 mins
Writing Time:	180 mins
Total Duration:	190 mins

NUMBER OF QUESTIONS: 5 TOTAL MARKS: 100

Instructions

- Answer ALL questions.
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.

Materials

- 1 Blue books are provided.
- Calculators are NOT permitted.
- Closed book examination.

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

1. Consider a Continuous-time Markov Chain (CTMC) with the following transition matrix

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

   (a) Write down and solve the equilibrium equations for this CTMC.

   (b) Write down the probability transition matrix P^j corresponding to the jump chain of the above CTMC.

   (c) Find the equilibrium distribution of P^j.

   (d) Do the two equilibrium distributions calculated differ? Comment on why this is the case. Is this unusual?

[15 marks]

Solutions:

To be added.

2. Consider an arrival process that can be in three separate phases. In each phase single arrivals occur after an exponentially distributed amount of time with rates 1.0, 2.0, and 3,0 respectively. No phase transitions occur when an arrival occurs. Transitions can occur between phase 1 and phase 2, with rate 10.0; between phase 2 and phase 3, with rate 5.0; and between phase 3 and phase 1 with rate 1.0. All of these transitions are exponentially distributed. This system is a Markov Process.

   (a) Write down the Q matrix that describes the arrival process. Do this in terms of the matrices that form Q.

   (b) What is the stationary distribution of being in the different phases of this Markov Process?

   (c) Determine the fundamental arrival rate of this process.

[10 marks]

Solutions:

To be added.

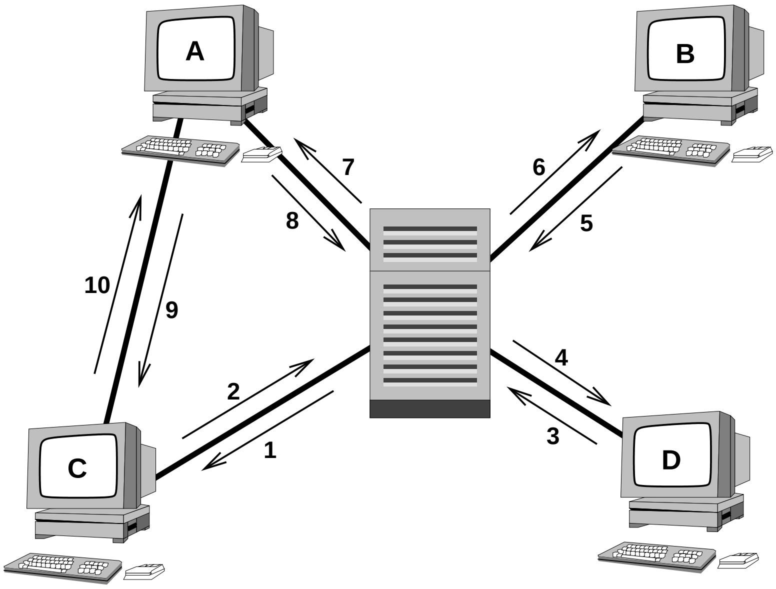

Figure 1: Simple traffic matrix problem. The numbered arrows denote the link labels.

3. Assume we are given the network shown in Figure 1, and told that that network uses shortest path routing, with the path length given by the number of hops. The 5 links in the network are bi-directional, and we label each direction as shown in Figure 1 with numerical labels from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the 12 possible end-to-end routes are listed in order as $(AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC)$.

   (a) A partial routing matrix A for the network in Figure 1, is given below. Complete the values for the row consisting of X's, and explain the meaning of this row.

$$
A = \begin{array}{c}
\begin{array}{cccccccccccc} AB & AC & AD & BA & BC & BD & CA & CB & CD & DA & DB & DC \end{array} \\
\left(\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
X & X & X & X & X & X & X & X & X & X & X & X \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{array}
\begin{array}{l}
\\
\text{link 1} \\ \text{link 2} \\ \text{link 3} \\ \text{link 4} \\ \text{link 5} \\ \text{link 6} \\ \text{link 7} \\ \text{link 8} \\ \text{link 9} \\ \text{link 10}
\end{array}
$$

   (b) In this network, imagine we have SNMP measurements of the traffic on each link, but we wish to estimate the traffic between end-points (assuming that traffic from a node to itself is zero). Express this problem as a linear inverse problem, and explain why the problem might be hard to solve.

(c) Given a traffic matrix describing the traffic between end-points defined by

$$\mathbf{T} = \begin{pmatrix} 0 & T_{AB} & T_{AC} & T_{AD} \\ T_{BA} & 0 & T_{BC} & T_{BD} \\ T_{CA} & T_{CB} & 0 & T_{CD} \\ T_{DA} & T_{DB} & T_{DC} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 5 & 6 \\ 6 & 0 & 10 & 12 \\ 9 & 12 & 0 & 18 \\ 12 & 16 & 20 & 0 \end{pmatrix}$$

write the traffic matrix as a column vector (using the same ordering as used in the routing matrix) and then determine how much traffic there would be on link 3.

(d) Determine from $\mathbf{T}$ the total volume of traffic into, and out-of the network at each end point T_A^{in} and T_A^{out} respectively, and then compute the traffic matrix we would estimate from a simple gravity model.

(e) We note that the results of the gravity model aren't the same as the actual traffic matrix. Explain (in a few sentences) a general approach to including the information from the link measurements to refine the accuracy of this estimate. [Note, you do not need to provide detailed mathematics of the approach used, only an intuitive description of this approach].

[20 marks]

Solutions:

(a) The routing matrix is a matrix showing the proportion of traffic from each end-to-end route, on each link. In the case specified, the entries are all zero or one, as there is no load sharing between alternate paths, and the complete routing matrix is

$$A = \begin{array}{c} \begin{array}{cccccccccccc} AB & AC & AD & BA & BC & BD & CA & CB & CD & DA & DB & DC \end{array} \\ \left(\begin{array}{cccccccccccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array} \begin{array}{l} \\ \text{link 1} \\ \text{link 2} \\ \text{link 3} \\ \text{link 4} \\ \text{link 5} \\ \text{link 6} \\ \text{link 7} \\ \text{link 8} \\ \text{link 9} \\ \text{link 10} \end{array}$$

The specified row indicates which end-to-end routes use link 5.

(b) If we write the traffic matrix as a vector of elements (using the same ordering as in the traffic matrix above), e.g. $\mathbf{x}^T = (T_{AB}, T_{AC}, T_{AD}, T_{BA}, T_{BC}, T_{BD}, T_{CA}, T_{CB}, T_{CD}, T_{DA}, T_{DB}, T_{DC})$, where, for example T_{AB} denotes the traffic from A to B, and the link traffic measurements are denoted by the column vector $\mathbf{y}$, then

$$\mathbf{y} = A\mathbf{x}$$

describes the linear relationship between $\mathbf{x}$ and $\mathbf{y}$. Given SNMP link measurements we know $\mathbf{y}$ and wish to estimate $\mathbf{x}$, and this is called a linear inverse problem. It may be hard to solve because there are more unknowns (12 end-to-end demands) than there are measurements (10 link measurements).

(c) The traffic matrix written as a column vector is

$$\begin{aligned}\mathbf{x}^T &= (T_{AB}, T_{AC}, T_{AD}, T_{BA}, T_{BC}, T_{BD}, T_{CA}, T_{CB}, T_{CD}, T_{DA}, T_{DB}, T_{DC}) \\ &= (4, 5, 6, 6, 10, 12, 9, 12, 18, 12, 16, 20)\end{aligned}$$

We can obtain the link traffic measurements by using the equations $\mathbf{y} = A\mathbf{x}$, but we only need to do this for one row of A corresponding to link 3, so we need to compute

$$y_3 = (0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0).(4, 5, 6, 6, 10, 12, 9, 12, 18, 12, 16, 20) = 6+12+18 = 36$$

(d) The traffic into the network at a end-point is given by the row sum of the matrix $\mathbf{T}$, and the traffic out of the network is given by the column sums, so

$$\mathbf{t}_{\text{in}} = \begin{pmatrix} T_A^{\text{in}} \\ T_B^{\text{in}} \\ T_C^{\text{in}} \\ T_D^{\text{in}} \end{pmatrix} = \begin{pmatrix} 15 \\ 28 \\ 39 \\ 48 \end{pmatrix} \quad \text{and} \quad \mathbf{t}_{\text{out}} = \begin{pmatrix} T_A^{\text{out}} \\ T_B^{\text{out}} \\ T_C^{\text{out}} \\ T_D^{\text{out}} \end{pmatrix} = \begin{pmatrix} 27 \\ 32 \\ 35 \\ 36 \end{pmatrix}$$

The simply gravity model approximates the traffic from X to Y by

$$T_{XY} \propto T_X^{\text{in}} T_Y^{\text{out}}$$

where T is the total traffic summed over all sources and destinations. We can calculate the constant of proportionality by noting that the total volume of traffic must be constant (in this case, the total volume is given by 130. We can actually express the entire traffic matrix as a vector product, e.g.

$$T = \frac{\mathbf{t}_{\text{in}} \mathbf{t}_{\text{out}}^T}{130} = \begin{pmatrix} \frac{405}{130} & \frac{480}{130} & \frac{525}{130} & \frac{540}{130} \\ \frac{756}{130} & \frac{896}{130} & \frac{980}{130} & \frac{1008}{130} \\ \frac{1053}{130} & \frac{1248}{130} & \frac{1365}{130} & \frac{1404}{130} \\ \frac{1296}{130} & \frac{1536}{130} & \frac{1680}{130} & \frac{1728}{130} \end{pmatrix}$$

(e) Note first that assuming no measurement noise, the problem we face arise because the set of equations $\mathbf{y} = A\mathbf{x}$ is underconstrained. Hence there are many possible solutions to these equations. We can refine the gravity model using the link measurements by using it to choose the solution that satisfies $\mathbf{y} = A\mathbf{x}$ that is "closest" to the gravity model solution. The term closest can be defined in multiple ways, but using a metric such as the Kullback-Libler divergence proves to work well. In practice, one may find that measurement noise leads to us having to solve a set of equations

$$\mathbf{y} = A\mathbf{x} + \mathbf{z}$$

which we can best accomplish by approximating the above approach with an optimization problem, minimize

$$||\mathbf{y} - A\mathbf{x}||^2 + \lambda^2 J(\mathbf{x})$$

where the penalty function J penalizes solutions that lie further away away from the gravity model.

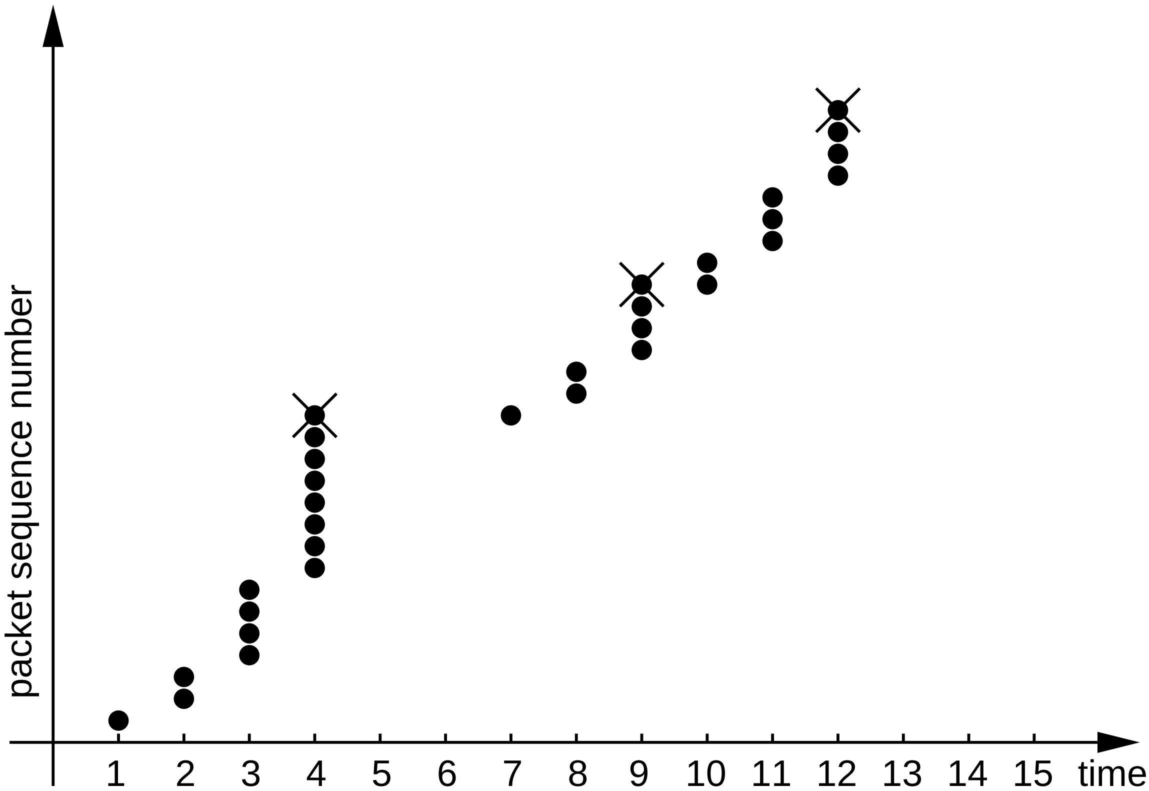

Figure 2: TCP congestion control example.

4. (a) Figure 2 shows a stream of packets from a single TCP source. The graph shows the packet sequence number vs the time at which the packet is sent. Crosses indicate packets that are dropped in transit.

    1. Draw graphs showing the size of the congestion window (in packets) at each time interval, and similarly the size of the slow-start threshold, assuming that this starts at ∞.
    2. Assuming TCP Reno is used, describe what is happening at each time point, and why it happens.

   (b) Assume that packet losses occur randomly, with each packet being lost with probability p, independent of the other packets, and the window size. Model the window size in the congestion avoidance phase by a series of probabilistic recurrence equations, and from this, determine a simple relationship between the loss probality and the window size during congestion avoidance (for greedy traffic sources, and small loss probability p). Assuming a round-trip time of R, and an average segment size of M, determine the average throughput of such a connection.

   (c) What effect might delayed acknowledgements have on the above results.

(d) Assume that the average throughput on a link is given by the law

$$x = \frac{M}{R\sqrt{p}}$$

where M is the average size of packet, and R is the round-trip time. Assume that he loss rate on the link is given by the approximation

$$p = \frac{[x-c]^+}{x}$$

where $[\cdot]^+$ denotes the positive part. Now determine the packet loss probability one might expect in equilibrium by using a fixed-point equation.

Solutions:

(a) Figure 3 shows the congestion window and Figure 4 shows the slow start threshold. See the following question for an explanation.

(b) The following occur at each of these time intervals

1. The TCP source starts with an initial window of one packet.
2. The slow start threshold is higher than the window size, so the process is in the slow start phase, where the window doubles each round-trip time, so in the second round-trip time, the window doubles to 2, and 2 packets are sent.
3. Similar to time interval 2, the window size doubles.
4. Similar to time interval 2-3, the window size doubles, but this time, too many packets are sent, so one is dropped.
7. The dropped packet is not detected until a timeout occurs roughly 3 round-trip times after the dropped packet. When the timeout occurs the protocol reduces the congestion window back to the initial window size, and retransmits the packet that was dropped. Also, the slow start threshold is reduced to half the window size.

8 More slow start (window doubling).

9. Similar to 8, but a packet is again dropped. In this case the packet loss is detected quickly through a triple duplicate acknowledgment, and the sources goes into fast retransmission/fast recovery, where the slow start threshold is set to half the window size, and the window size is only dropped by half as well.
10. From 9. the window size is 2, and the slow-start threshold is 2, so the process is in congestion avoidance, and the window size grows by 1 each round-trip.
11. More congestion avoidance.
12. and so on.

(c) Model the window size as a discrete time process w_n, when n denotes the number of round-trip times. In equilibrium, we model the congestion avoidance by increasing the window by one each round-trip when no packet is lost, and decreasing it by a factor of 2 whenever a packet is lost, i.e.

$$w_{n+1} = \begin{cases} w_n + 1, & \text{if no packets are lost,} \\ w_n/2, & \text{if any packets are lost.} \end{cases}$$

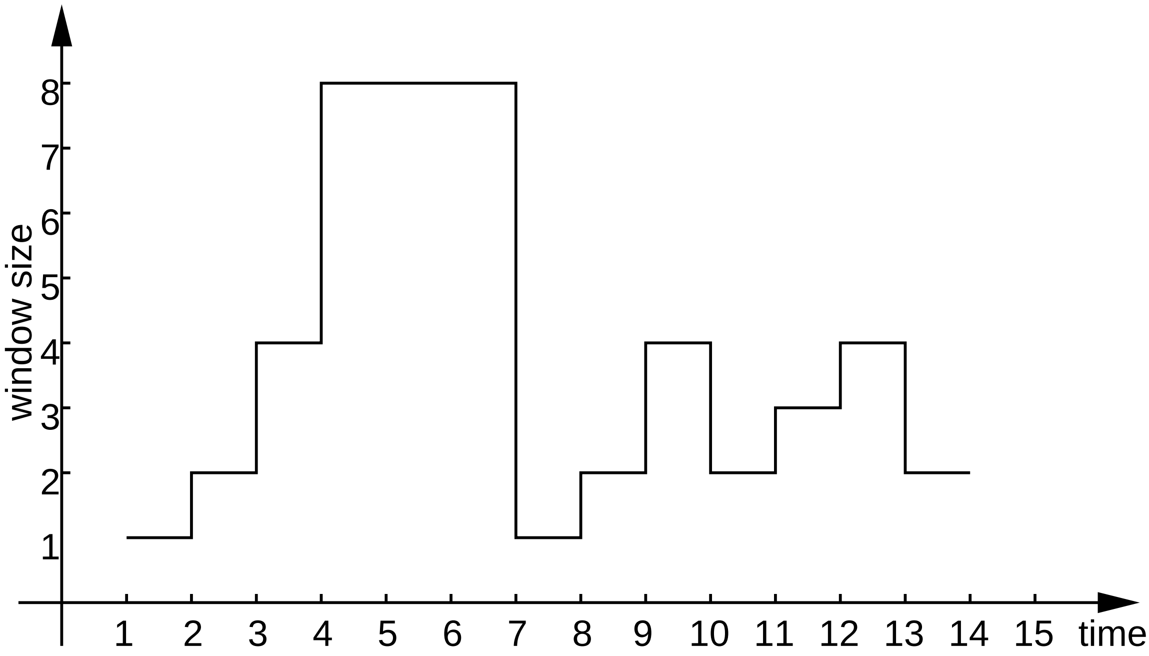

Figure 3: TCP congestion control example.

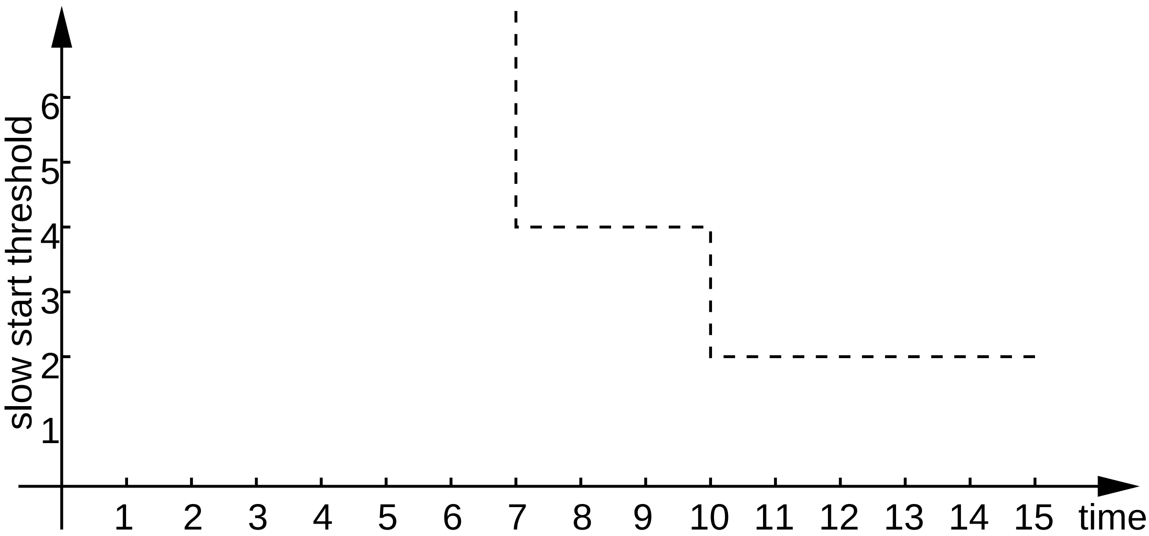

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Figure 4: TCP congestion control example.

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The probability a packet no packet is lost during a particular round-trip time is $(1-p)^{w_n}$, which we can approximate for small p by $1 - w_n p$. Hence, we model w_n by the recurrence

$$w_{n+1} = \begin{cases} w_n + 1, & \text{with probability } 1 - w_n p, \\ w_n/2, & \text{with probability } w_n p. \end{cases}$$

The expected value of w_n is therefore

$$\begin{aligned} E[w_{n+1}] &= p\{\text{packet loss}\} w_n/2 + p\{\text{no packet loss}\}(w_n + 1) \\ &= w_n p w_n/2 + (1 - w_n p)(w_n + 1) \\ &= 1 + w_n - w_n p - w_n^2 p/2. \end{aligned}$$

In equilibrium, $\bar{w} = E[w_{n+1}] = E[w_n]$ so

$$\begin{aligned} \bar{w} &= 1 + \bar{w} - \bar{w}p - \bar{w}^2 p/2 \\ \bar{w}^2 p/2 + \bar{w}p &= 1. \end{aligned}$$

As p is assumed to be small and $\bar{w} > 1$, we can ignore the $\bar{w}p$ term, and so

$$\bar{w} \simeq \sqrt{\frac{2}{p}}.$$

If the average packet size is M, and the average round-trip time is R, then the average throughput rate will be

$$x = \frac{M}{R}\sqrt{\frac{2}{p}}.$$

(d) Delayed acknowledgements occur when a receiver delays sending the ackowledgement for a packet to allow piggy backing of acknowledgements on returning data packets, or to allow multiple packets to be acknowledged at once. For a greedy source, typically this results in one acknowlegement for each group of b packets. Delayed acknowledgements therefore reduce the rate at which the sender receives acknowledgements, and therefore the rate of increase of the window size by a factor b, so that the recurrence relation becomes

(e) We know that TCP responds to loss indications, so in equilibrium we would expect $p > 0$, and hence, for the model presented, $x > c$. Thus we can write the two equations as

$$\begin{aligned} x &= \frac{M}{R\sqrt{p}} \\ p &= \frac{x - c}{x}. \end{aligned}$$

We can combine these into one equation by substituting the first into the second, to get

$$p = \frac{\frac{M}{R\sqrt{p}} - c}{\frac{M}{R\sqrt{p}}}.$$

We see *fixed-points* of this equation, e.g. points p^* such that the above equation is true. In general we could find such a solution by repeated substitution, but in this particular case we can solve the equation into a quadratic and solve directly to get p^*.

5. Write an essay on **one** of the following two topics:

   (a) Describe the key principles underlying TCP's operation, in particular, describe briefly the goal, and approach of the congestion control, but also note the end-to-end principle, and robustness principle, and describe.

   (b) What are do you need to get right in modelling Internet traffic? In particular, how does measurement effect modelling? What principles need to be applied when building models? Is a model automatically better if it fits a dataset better? What features are commonly observed in Internet traffic and why are they important?

   Your essay should be no longer than one page, and clear, concise presentation will be taken into account in marking. [20 marks]

   Solution: Obviously no correct solution exists to an essay question. In marking the question I will take into account the

   - clarity of writing, and quality of presentation,
   - a well described motivation, and
   - concise but insightful results.

   Note the page limit, and stick to it.