Duality rules

Matthew Roughan

July 12, 2017

Duality rules

	Primal	Dual
1.	$\max z = \sum_{j=1}^{n} c_j x_j + z_0$	$\min w = \sum_{i=1}^{m} y_i b_i + z_0$
2.	$\sum_{j=1}^{n} a_{ij} x_j = b_i$	y_i free
3.	$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$	$y_i \geq 0$
4.	$\sum_{j=1}^{n} a_{ij} x_j \geq b_i$	$y_i \leq 0$
5.	$x_j \geq 0$	$\sum_{i=1}^{m} y_i a_{ij} \geq c_j$
6.	$x_j \leq 0$	$\sum_{i=1}^{m} y_i a_{ij} \leq c_j$
7.	x_j free	$\sum_{i=1}^{m} y_i a_{ij} = c_j$

Complementary Slackness Relationships: At optimal solutions $\mathbf{x}$ for (P) and $\mathbf{y}$ for (D), the following relationships must hold:

1. $x_j \left(\sum_{i=1}^{m} y_i a_{ij} - c_j \right) = 0, \quad j = 1, \ldots, n.$
2. $\left(b_i - \sum_{j=1}^{n} a_{ij} x_j \right) y_i = 0, \quad i = 1, \ldots, m.$

where

1. is redundant if $\sum_{i=1}^{m} y_i a_{ij} = c_j$ in (D), *i.e.,* the x_i are free.
2. is redundant if $\sum_{j=1}^{n} a_{ij} x_j = b_i$, in (P), *i.e.,* the LP has equalities.