Assignment 5: Due Monday Oct 7th at 3pm.

Assignments to be handed in through MyUni. Please ensure written assignments are clearly legible. Typed assignments are preferred. Some help may be given in practicals to help get you started with Overleaf/LaTeX in order to present your work well.

1. The knapsack problem is the problem of choosing which items to put in a knapsack (or backpack). Each item has value, but you have a limited volume in the pack. An example problem with total value z and maximum volumne 45 is given below.

$$\begin{array}{llcrcrcrcrcrcrcl} \text{max.} & z & = & 43x_1 & + & 41x_2 & + & 27x_3 & + & 32x_4 & + & 15x_5 & + & 50x_6 & & \\ \text{s.t.} & & & 20x_1 & + & 19x_2 & + & 14x_3 & + & 16x_4 & + & 7x_5 & + & 28x_6 & \leq & 45 \\ & & & & & & & & & & & & & & & \end{array}$$
$$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6 \ = \ 0, 1$$

Solve and answer these questions:

(a) Relax the (0–1) integrality condition to simple non-negativity and solve. You should be able to solve this by inspection, but you may use AMPL.

(b) Relax the integrality condition to $0 \leq x_i \leq 1$ for all i, and solve using AMPL.

(c) Now solve the true (ILP) with AMPL. Please include a copy of your AMPL code with your solution.

(d) Comment on the relative size of the objective function for each solution, and why your observation should be intuitive.

(e) Does rounding the solutions in (i) and (ii) give the optimal binary solution from (iii)?

(f) Now apply a greedy algorithm to the problem. How does it compare to the optimal solution?

(g) Calculate the computation complexity of the greedy approach in Big-O notation. Assume that any sorting is done using “mergesort”.

[20 marks]