Network-Design Sensitivity Analysis

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The Problem

Traffic Matrices

- simply a matrix of traffic from $A \rightarrow B$
- fundamental input for most network planning (invariant)

But good network data are notoriously hard to get, and inaccurate

- measurements are an afterthought
  - often you don’t get what you would like
  - measurements aren’t calibrated
  - missing data is a big issue

- big data
  - sampling, sketching, ...

- prediction
  - planning needs predictions, which have errors
  - what about green fields planning?
Existing Network Planning Solutions

- Ignore the issue, and make a guess
- Make a guess, and then add 50%
- Oblivious routing
  - routing scheme that works for any traffic matrix
- Valiant network design
  - network design that works for any traffic matrix

Are any of these used?
Valiant network design [Val82, ZSM04, ZSM05]

backbone node

access network
Valiant network design

Abstract the access network to have capacity $C$
Valiant network design

Simple case (which can be generalized)

- don’t know the traffic matrix $t_{pq}$
- assume access capacity $C$ to each backbone node
- this limits traffic matrix

$$\sum_q t_{pq} \leq C \quad \text{and} \quad \sum_p t_{pq} \leq C$$

- route traffic demand $t_{pq}$ as follows
  - divide it into $|N|$ even groups
  - route group $i$ as follows $p \rightarrow i \rightarrow q$
  - load balance across all of the possible 2 hop routes
  - do the same for all $p, q \in N$
Valiant network design
Valiant network design

- compare to direct routing
  - each packet traverses 2 hops
  - $2 \times$ the bandwidth needed over optimal (a star)
- but it is *oblivious* to the traffic matrix
  - this design is provably the best oblivious network design [ZSM05] (given a certain cost model).
- it also has great advantages for survivability
  - can survive any combination of node failures
  - highly robust to link failures as well
  - only need marginal increases in link capacities
Better yet

- both of the above approaches assume we don’t know the traffic matrix
  - they are **oblivious**
  - but that has a cost in terms of efficiency
- but in reality we know something
  - e.g. SNMP measurements of traffic on links
  - e.g. partial netflow across network
- **can we design a network using the information we have, but taking into account the information we are missing?**
  - obviously we can, but how?
Mean-Risk Analysis in Finance

- Reduce volatility (and hence risk) of a portfolio by including multiple “uncorrelated” stocks
- Overall risk is reduced by balanced portfolio
  - no such thing as a free lunch
  - lowers returns if we knew the future
  - but in absence of predictions, we are overall better off
Imagine we need to carry the traffic $t_{i,j}$

- optimal capacities

$$c_{i,j} = t_{i,j}$$
Now assume we don’t know the $t_{i,j}$ exactly

\[ c_{i,j} = \hat{t}_{i,j} + \gamma \sigma_{i,j} \]

- $\hat{t}_{i,j}$ is predicted traffic
- $\sigma_{i,j}$ is some estimate of possible errors
- $\gamma$ is an over-build factor
Issues

- We can often get predictions $\hat{t}_{i,j}$
- Estimating errors $\sigma_{i,j}$ in predictions is harder
- Choosing $\gamma$ is hard
  - it balances risk against efficiency
  - it’s hard to choose because the balance is poor here
So let's build it more like this

- It's "less optimal" in one sense
  - We have to build more links
  - But shorter links are usually cheaper
- The one long link multiplexes the traffic from left to right
  - Capacity on long link

\[ C = \sum_{i,j} \hat{t}_{i,j} + \gamma \sigma \]
Assuming independent errors

\[ \sigma \neq \sum_{i,j} \sigma_{i,j} \]

To determine \( \sigma \) we need an error model

- very typically, people use IID Gaussian

\[ \sigma^2 = \sum_{i,j} \sigma_{i,j}^2 \]

e.g. if \( \sigma_{i,j} \) above were 2

\[ \sigma = \sqrt{4 + 4 + 4 + 4} = 4 = \frac{1}{2} \sum_{i,j} \sigma_{i,j} \]

So when errors are large enough

\[ C = \sum_{i,j} \hat{t}_{i,j} + \gamma \sigma < \sum_{i,j} c_{i,j} \]
Multiplexing gain

- The phenomena is called **multiplexing gain**
  - its been well known for a long time
  - but it doesn’t seem to be used (explicitly) in IP network design?
- The analogy with finance is clear
  - a portfolio decreases risk by including shares whose risks are (hopefully) uncorrelated, so total risk is less
  - multiplexing does the same
- There is a cost for lack of knowledge
  - it doesn’t have to be too bad
  - but you don’t want to ignore data
Networks

Networks are more complicated than the above example but same deal applies

- optimal when traffic is known isn’t robust (its sensitive)
- optimizing separately is a bad idea
- so some aggregation should happen
Tricky bits

The goal to balance risk with optimality

- What is risk here
  - is IID Gaussian a good model for errors?
  - how do we measure risk?

- What is optimal
  - lots of work on network design, so we will use a simple case

- How do you balance them?
  - stochastic optimization
  - but still need a hook?
  - we will do it using an ensemble of synthetic traffic matrices
Traffic Matrix Synthesis 101

- Simplest idea is IID, but Gaussian doesn’t work
  - Log-normal [NST05]
    - reasonable match to observed distribution
    - doesn’t have any structure

- Gravity model [Rou05], e.g.
  - generate “populations” $p_i$
  - traffic $t_{i,j}$
    \[ t_{i,j} \propto p_i p_j \]
  - matches some structure, and distribution
  - certainly isn’t perfect

- Not a lot of other research on the topic
  - and we want to do something slightly different anyway
  - we don’t want a completely random ensemble
Our goal

Generate an ensemble of TMs “like” a predicted matrix

- **admissible**
  - satisfies constraints
    - non-negative
    - imposed by network

- **centered**
  - their average centers on the predicted matrix

- **controlled**
  - variance around the predicted matrix can be controlled
  - linear parameter $\beta$
  - similar to the role of $\sigma$ in Gaussian case
Typical methods

Form ensemble by adding noise $z_{i,j}$

- usually IID
- often Gaussian

\[
\text{Additive : } y_{i,j} = t_{i,j} + \sigma z_{i,j}, \\
\text{Multiplicative : } y_{i,j} = t_{i,j} \left(1 + \sigma z_{i,j}\right),
\]

Both have problems:

- IID loses any structure
- Allows negative values
  - can truncate, but this introduces 0s, and de-centers
- Scaling
  - multiplexing means estimates of large elements should be relatively more accurate
  - neither of these have the correct scaling
Start with admissibility: describe by constraints

- We use four sets of constraints
  1. **Non-negativity**: $t_{i,j} \geq 0, \forall i, j = 1, 2, \cdots, N,$
  2. **Row sums**: $\sum_j t_{i,j} = r_i, \forall i,$
  3. **Column sums**: $\sum_i t_{i,j} = c_j, \forall j,$ and
  4. **Total traffic**: $\sum_{i,j} t_{i,j} = \sum_i r_i = \sum_j c_j = T,$

- Chosen to be exemplars
  - Easier to measure/predict total in/out traffic at a PoP
  - Matched to previous work on inference

- Could have any convex constraints
Spherically Additive Noise Model (SANM)

Let’s enforce fundamental constraints by design

- Note that for non-negative traffic we can write it

\[ t_{i,j} = a_{i,j}^2 \]

- And total traffic constraints says

\[ \sum_{i,j} a_{i,j}^2 = T, \]

- So traffic matrix sits on a \( N^2 \) dimensional hyper-sphere
Spherically Additive Noise Model (SANM)

So, add noise in the $N^2$ dimensional space, along the hypersphere

- form new matrix
  \[ y_{i,j} = (a_{i,j} + \beta z_{i,j})^2 \]
- then scale back to hypersphere, i.e., like normalizing
  - we are adding noise for a point on the hyper-sphere (hence the name)
- use Iterative Proportional Fitting (IPF)
  - finds “closest” TM on the hyper-sphere that fits the constraints
Synthesis Analysis

![Graph showing MSRE(β) vs β]

- **Small β approximation**
- **Large β approximation**

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Network Design

We have looked at a few cases, but let's just take one here: redesign of Abilene.
Conclusion

- Good design needs robustness to errors in predictions
  - extreme case is oblivious, but this is wasteful
  - using a little bit of information can improve things
- Mechanism to do so is to be able to generate synthetic traffic matrices
  - Spherically Additive Noise Model
  - nice properties
  - seems to work in practice


Further reading II