Acoustic metamaterial chains involving inertial amplification: Linear case

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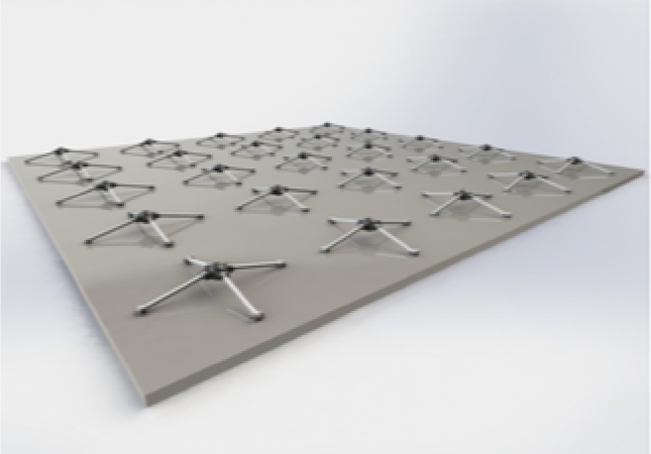
AMMs using lightweight attachments

General area

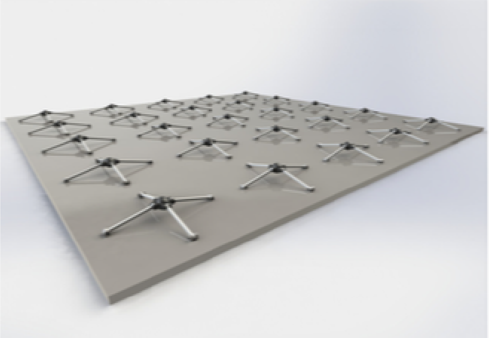
- Seek effective low-frequency noise isolation using AMMs.
  - Components that cause noise pollution.
- Without using heavy elements.
  - Heavy = impractical for applications.
- Use inerters
  - Inertial force proportional to relative motion.
  - E.g. J-dampers in F1 racing car suspension systems.

AMMs using lightweight attachments



Frandsen et al, J Appl Phys, 2016

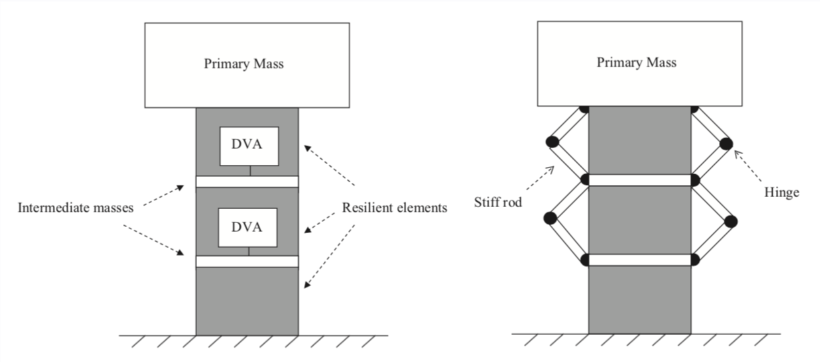
AMMs using lightweight attachments



Frandsen et al, J Appl Phys, 2016

"...band gaps that are exceedingly wide and deep ...as much as twenty times less added mass compared to what is needed in a standard local resonator configuration"

An application (Dylejko & MacGillvray, 2014)



Vibration isolators

- Prevent noise propagating through supporting structure.
- Suffer from high frequency internal resonances.
- Nb. different motivation.

AMM as 1D chain

Metamaterials — according to Wikipedia

"A material engineered to have a property that is not found in nature"

Model framework: 1D chain

- where something *"meta"* happens in the boxes.

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- No dissipation.

AMM as 1D chain

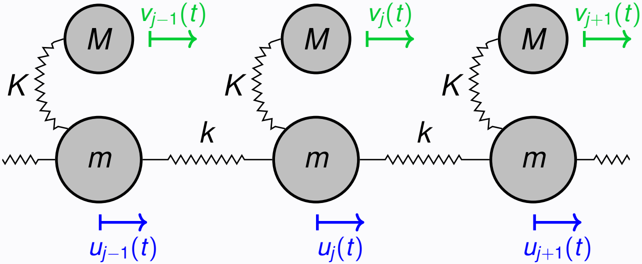
Metamaterials — according to Wikipedia

"A material engineered to have a property that is not found in nature"

Model framework: 1D chain

- where something *"meta"* happens in the boxes.
- No dissipation.
- Infinite chain “facilitates analysis".

Standard AMM: Mass-in-mass chain

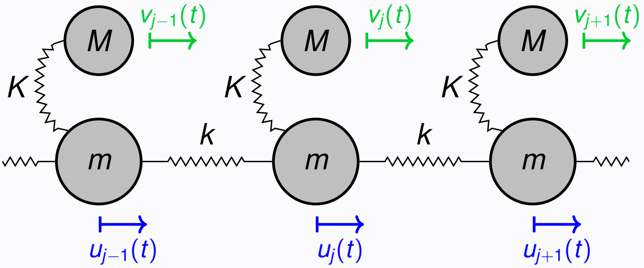


Equations of motion

$$m\,\ddot{u}_j + k\,(2\,u_j - u_{j-1} - u_{j+1}) + K(u_j - v_j) = 0$$

$$M\,\ddot{v}_j + K\,(v_j - u_j) = 0$$

Standard AMM: Mass-in-mass chain



Write

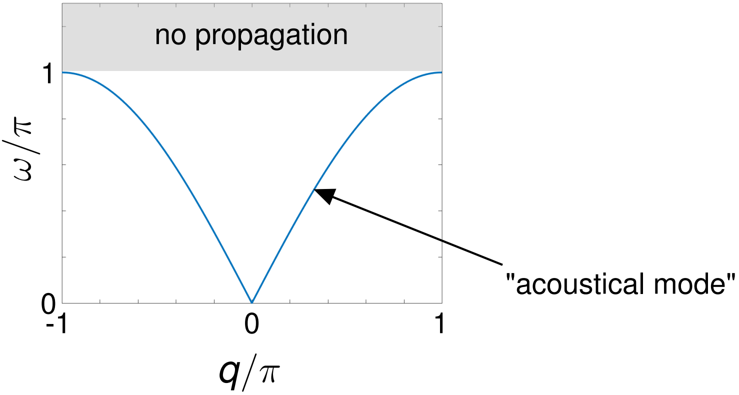
$$u_j = \hat{u}\,\mathrm{e}^{\mathrm{i}\,(q\,j-\omega\,t)} \quad \text{and} \quad v_j = \hat{v}\,\mathrm{e}^{\mathrm{i}\,(q\,j-\omega\,t)}$$

where $\omega =$ angular freq; $q =$Bloch–Floquet waveno $\in (-\pi, \pi]$.

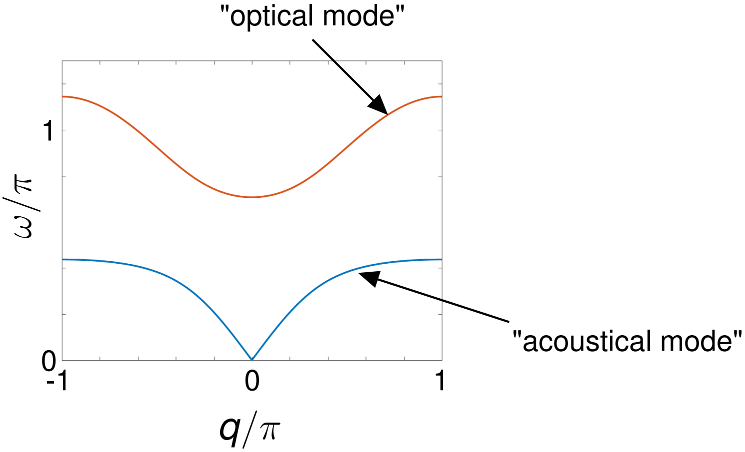
Dispersion relation

$$m\,M\,\omega^4 - \{K\,(m+M) + 2\,k\,(1-\cos q)\}\,\omega^2 + 2\,(1-\cos q) = 0$$

Band diagrams: $m = 1$; $M = 0$; $K = k = \pi^2/4$



Band diagrams: $m = 1$; $M = 1$; $K = k = \pi^2/4$



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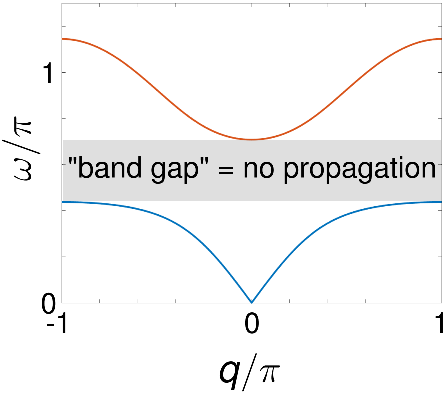

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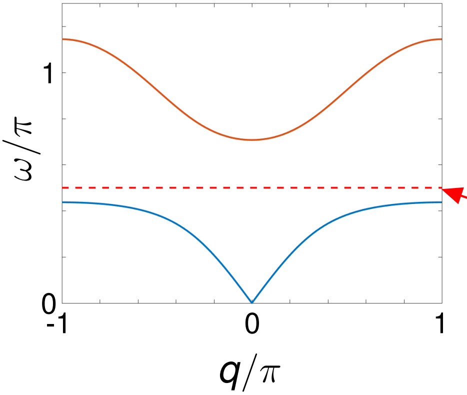
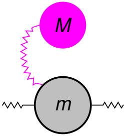


anti-resonance: $\omega^2 = \omega_*^2 \equiv \frac{K}{M}$



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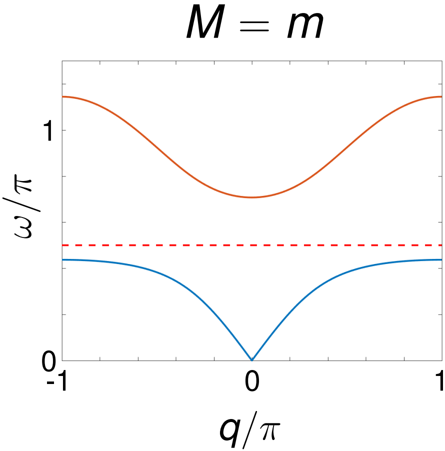

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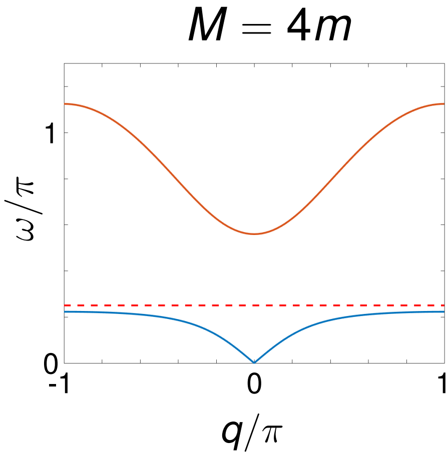
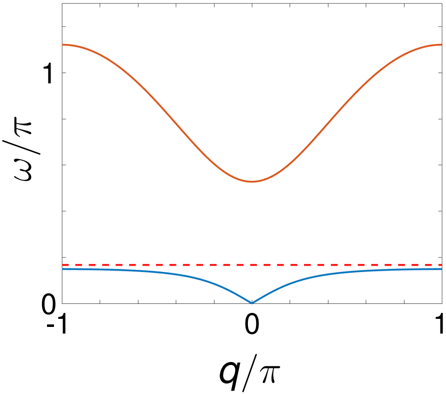


Lower resonances
= lower band gap

Band diagrams: $m = 1$; $K = k = \pi^2/4$

$M = 9m$

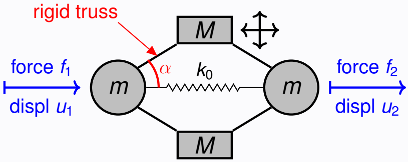


Lower resonances
= lower band gap

But mass increases

Basic element (the black box)

Lightweight attachments = geometry



Linearised equations of motion

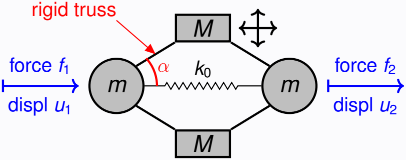
$$-f_1 = \omega^2 \left(m + \frac{M}{2}(1+\gamma^2) \right) u_1 + \frac{\omega^2 M}{2}(1-\gamma^2)\, u_2 + k_0(u_2 - u_1)$$

$$-f_2 = \frac{\omega^2 M}{2}(1-\gamma^2) u_1 + \omega^2 \left(m + \frac{M}{2}(1+\gamma^2) \right) u_2 + k_0(u_1 - u_2)$$

with geometrical parameter $\gamma = \cot \alpha$.

Basic element (the black box)

Lightweight attachments = geometry



Compliance and stiffness matrices

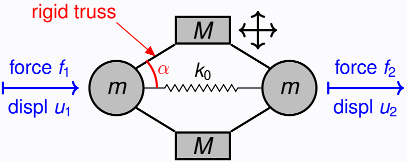
$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = C_0 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = D_0 \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

Resonance: $\omega = \omega_0$ st. $\det C_0 = 0$

Anti-resonance: $\omega = \omega_*$ st. $C_0 = \operatorname{diag} C_0, \; D_0 = \operatorname{diag} D_0$

Basic element (the black box)

Lightweight attachments = geometry



Compliance and stiffness matrices

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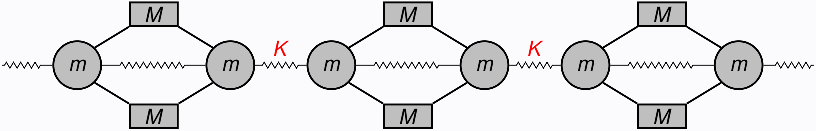
Resonance: $\omega_0^2 = \dfrac{2\,k_0\,(m+M)}{m^2 + m\,M\,(1+\gamma^2) + M^2\,\gamma^2}$

Anti-resonance: $\omega_*^2 = \dfrac{2\,k_0}{M\,(\gamma^2 - 1)}$ for $\gamma < 1, \quad (\alpha < \pi/4)$

Infinite chain

Connect basic elements with springs



Infinite chain

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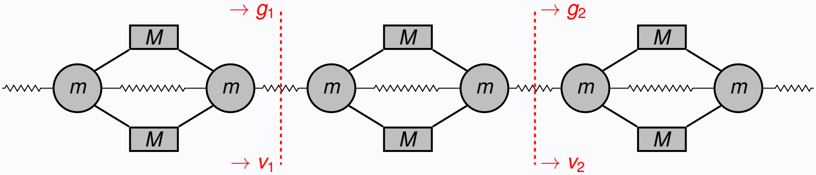


Transfer matrix P (encodes all info):

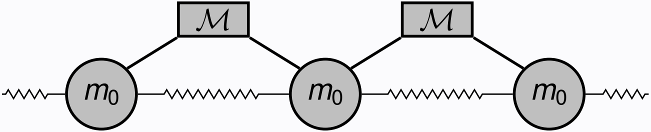
$$\begin{pmatrix} v_2 \\ g_2 \end{pmatrix} = P \begin{pmatrix} v_1 \\ -g_1 \end{pmatrix} \quad \text{with e'vals} \quad \lambda_\pm = \exp(\mathrm{i}\, q)$$

where $q =$ Bloch–Floquet wavenumber.

- ω_* unchanged but ω_0 is (in general).

Limit 1: Yilmaz & Hulbert (Phys Lett A, 2010)

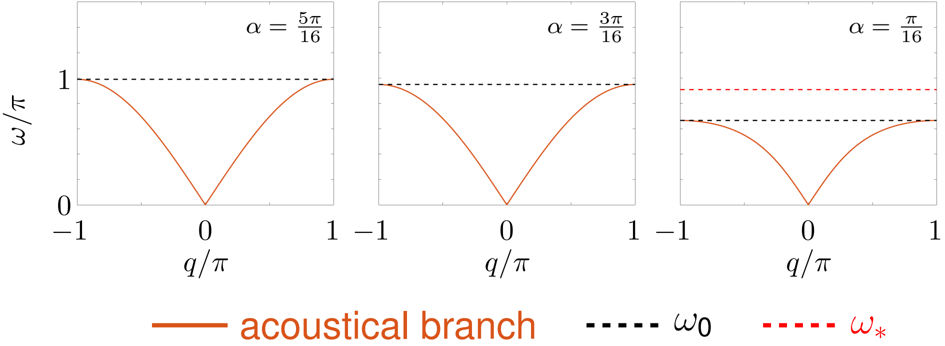
$K \to \infty$; $M = \mathcal{M}/2$; $m = m_0/2$



$$\omega_0^2 \to \frac{4\,k_0}{m_0 + \mathcal{M}\,\gamma^2} = \frac{2\,k_0}{m + M\,\gamma^2}$$

$$\omega_*^2 \to \frac{4\,k_0}{\mathcal{M}\,(\gamma^2 - 1)} = \frac{2\,k_0}{M\,(\gamma^2 - 1)} \quad \text{for} \quad \alpha < \pi/4.$$

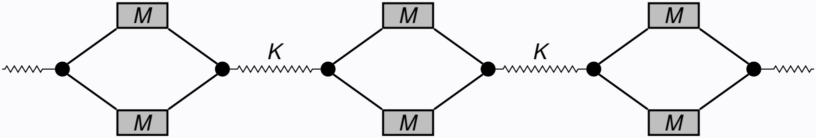
Limit 1: Band diagrams $k_0/m_0 = \pi^2/4$, $\mathcal{M}/m_0 = 0.1$



- Resonant frequency ω_0 controls band height.
- Anti-resonant frequency ω_* controls gap “depth".

Limit 2: Bobrovnitskii (Acoust Phys, 2014)

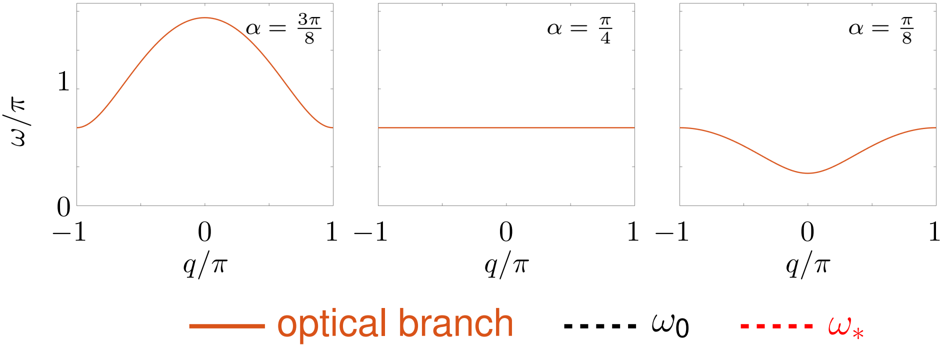
$k_0, m \to 0$



$$\omega_0^2 \to 0 \quad \text{and} \quad \omega_*^2 \to 0$$

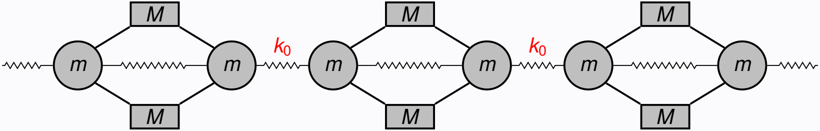
- Basic element has no restoring.
- Always have branch $\omega(q) = 0$.

Limit 1: Band diagrams $K/M = \pi^2/2$



- Acoustical branch (not shown) on q-axis.

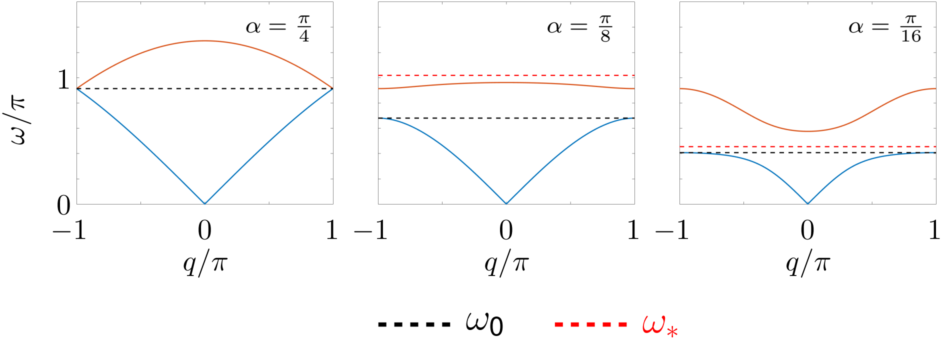
Back to original system with $K = k_0$



$$\omega_0^2 = \frac{2\,k_0}{m + M\,\gamma^2} \quad \text{and} \quad \omega_*^2 = \frac{2\,k_0}{M\,(\gamma^2 - 1)}$$

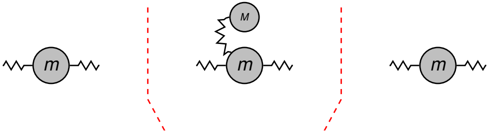
- Same resonance and anti-resonance as Yilmaz chain.
- Half the attached mass, but…

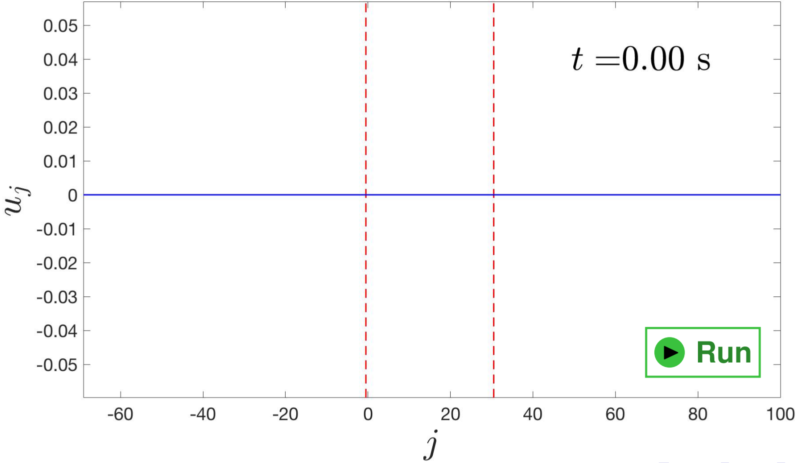
Band diagrams: $M/m = 0.2$; $K/M = \pi^2/2$



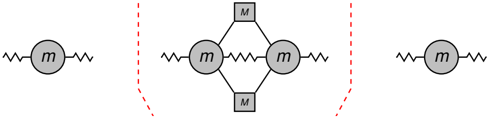
- …at expense of acoustical and optical branches.
- Anti-resonance ω_* controls concavity of optical branch.

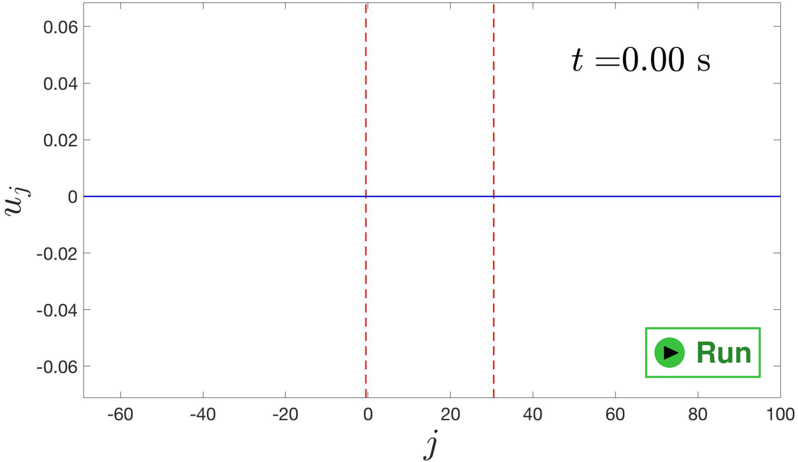
Mass-in-mass: $M/m = 0.1$; $\omega/\pi = 0.45$





Our system: $M/m = 0.1$; $\omega/\pi = 0.45$; $\alpha = \pi/16$





Final slide

Summary

- Toy model.
- Attached masses excite resonance/anti-resonance to suppress low-frequency vibrations.
- Lightweight attachments can achieve this using geometry.

At KOZWaves 2020...

Alex's idea:

- Nonlinearity (physical or geometrical) to transfer energy to higher frequencies (and damp).
- Modulational instability between acoustical and optical branches for efficient energy transfer.