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Using geometry to control bandgaps along acoustic metamaterial chains

Luke Bennetts, Uni Adelaide; Malte Peter, Uni Augsburg
Paul Dylejko, Alex Skvortsov, Maritime Division, DST Group, Melbourne

maths.adelaide.edu.au/luke.bennetts

@LukeBennettsUoA

Setting

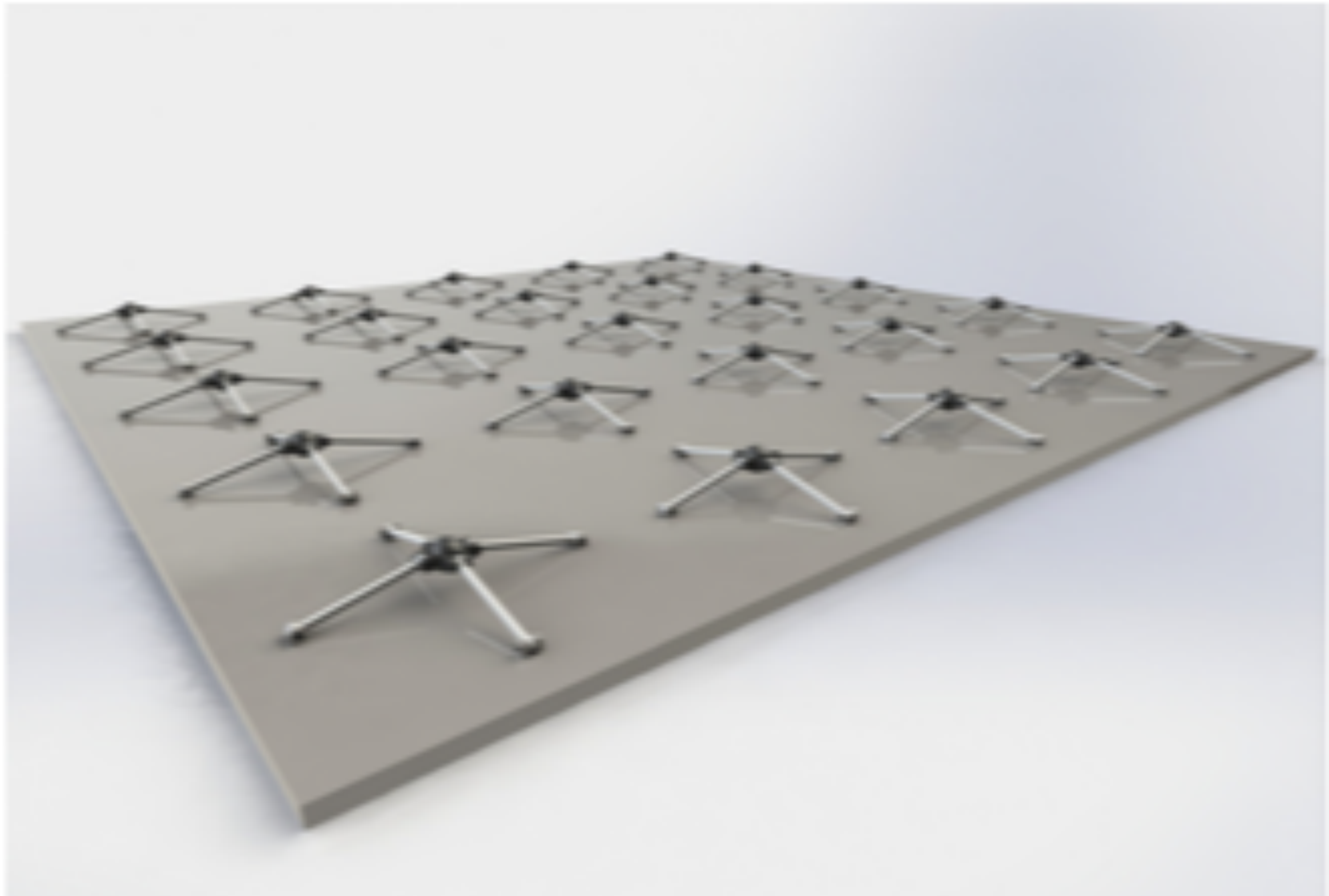
- Seek low-frequency structural vibration isolation
  - Using AMMs
  - Components that cause noise pollution
- Without using heavy elements
  - Heavy = impractical for applications

<u>Inertial amplification using geometry</u>

Two main groups working on this:

1. Yilmaz et al, Dept Mech Eng, Bogazici Uni, Istanbul
2. Jensen et al, Dept Mech Eng, DTU, Copenhagen

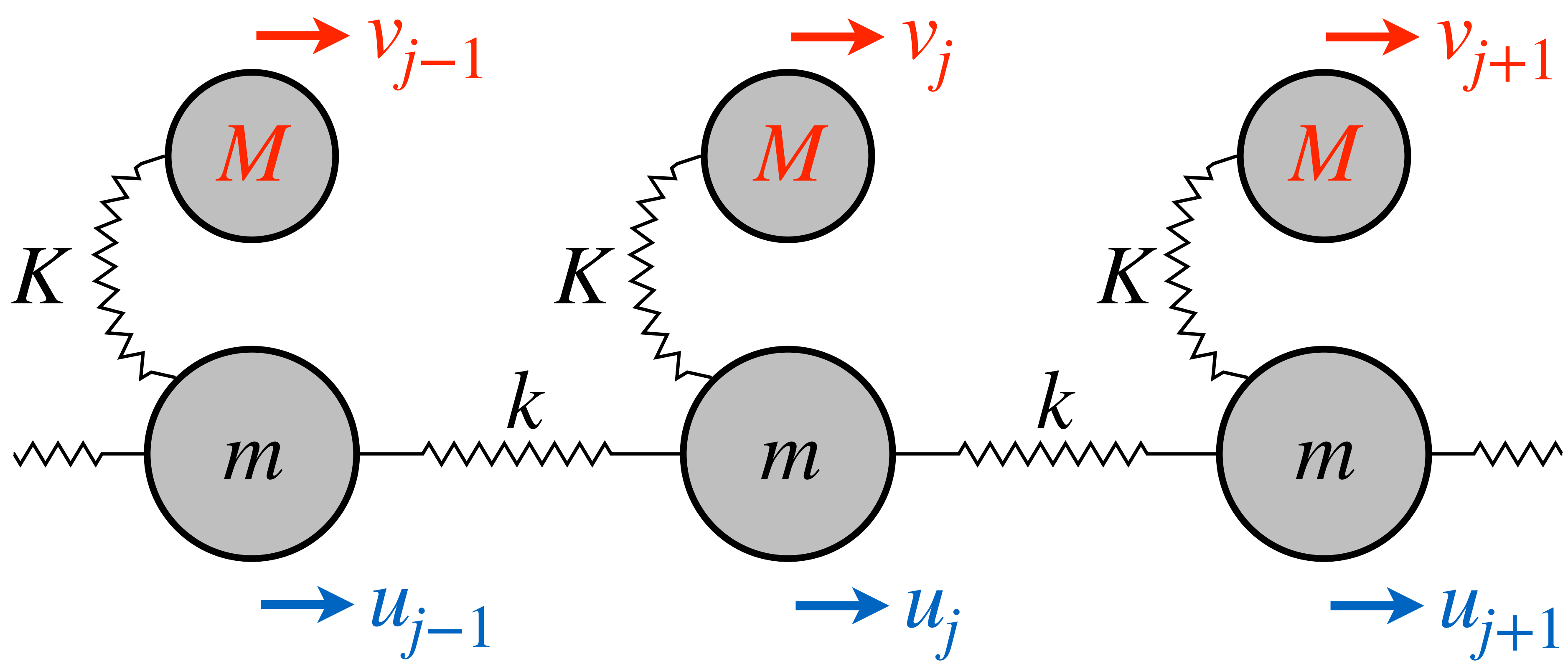
AMMs using lightweight attachments



Frandsen et al, J Appl Phys, 2016

"... band gaps that are exceedingly wide and deep ... as much as 20 times less added mass compared to what is needed in a standard local resonator configuration"

Mass-in-mass chain

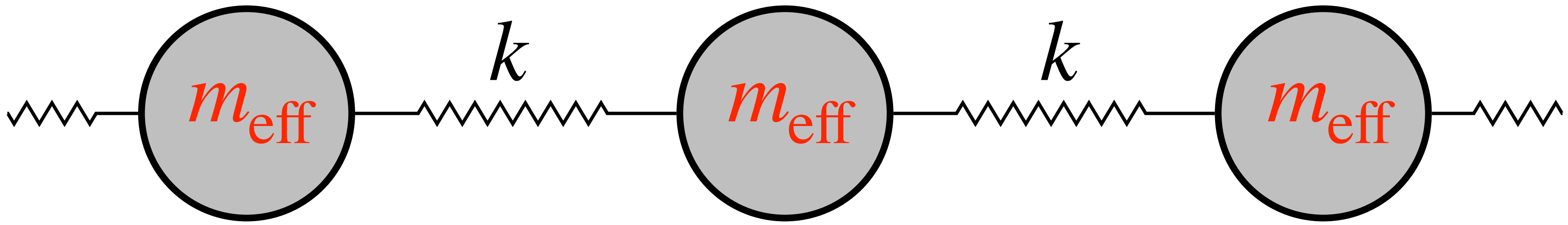


Equation of motions (jth masses):

$$m\,\ddot{u}_j + k\,(2\,u_j - u_{j-1} - u_{j+1}) + K(u_j - v_j) = 0$$

$$M\,\ddot{v}_j + K\,(v_j - u_j) = 0$$

Homogenised "effective" chain

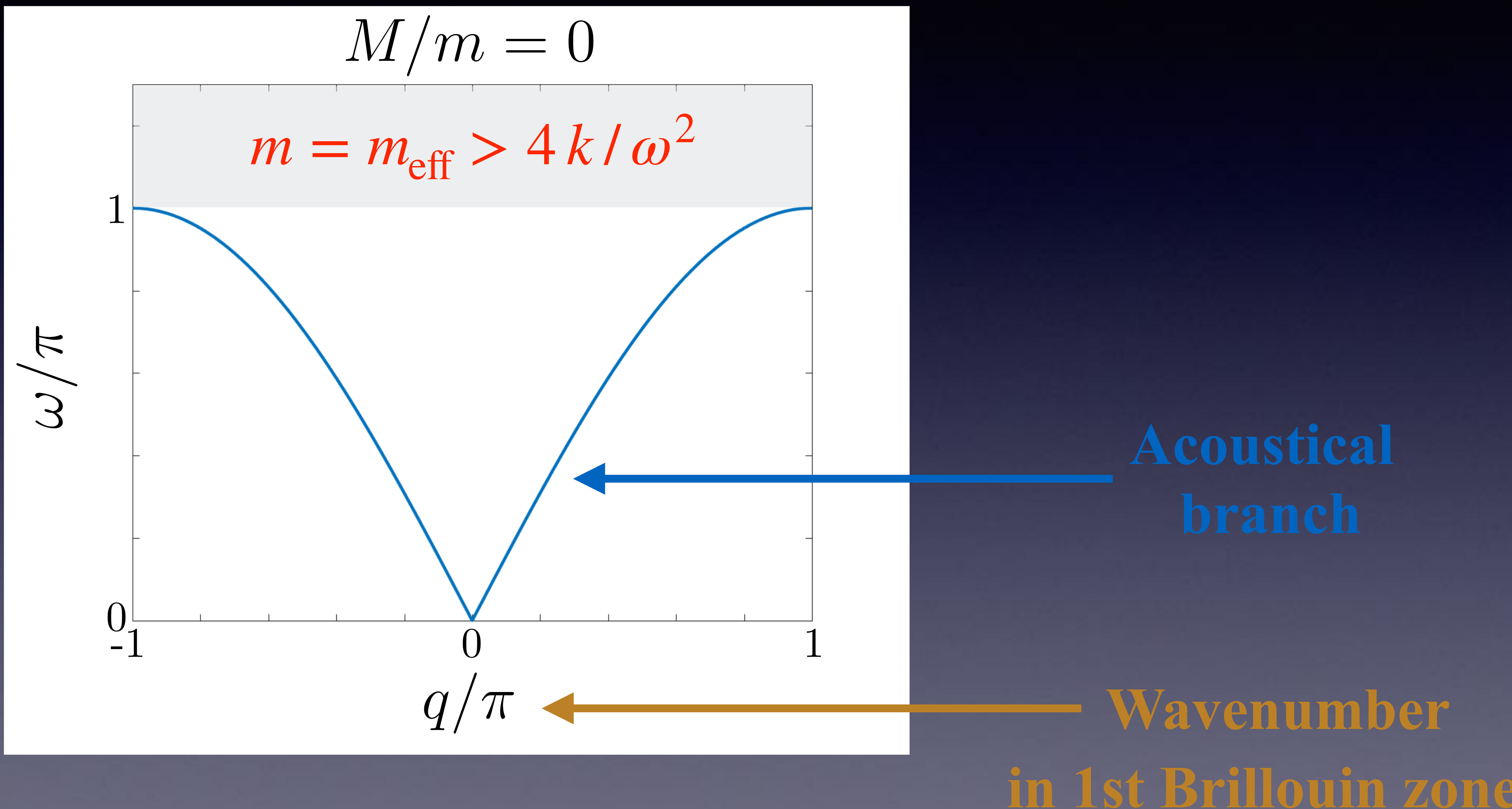


$$m_{\text{eff}} = m + \frac{\omega_*^2 M}{\omega_*^2 - \omega^2} \quad \text{and} \quad \underbrace{\omega_* = \sqrt{\frac{K}{M}}}_{\text{anti-resonance}}$$

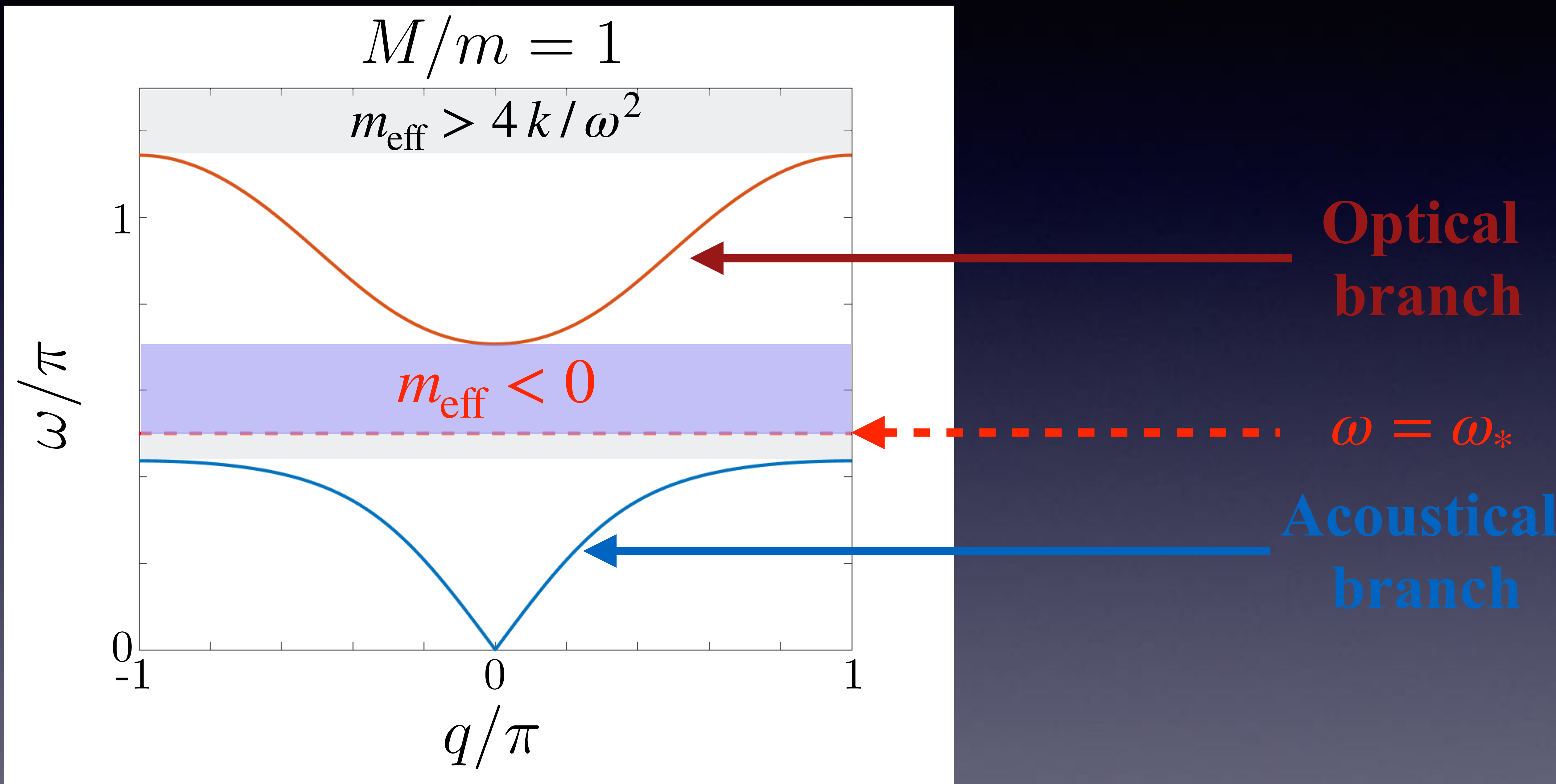
$m_{\text{eff}} \to \pm\infty$ as angular frequency $\omega \to \omega_*^{\mp}$

System is "inertially amplified" around ω_*

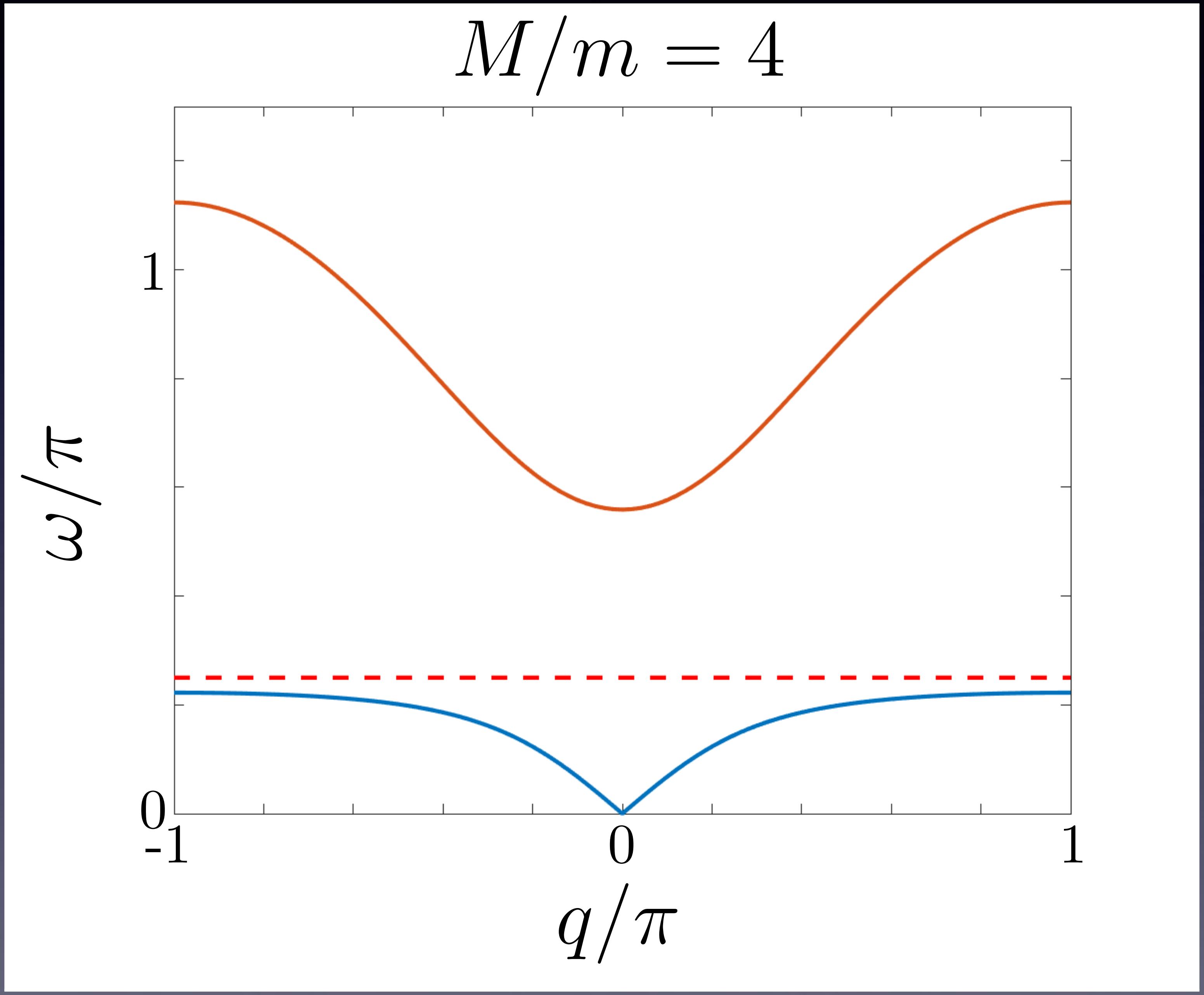
Band diagram: $k = K = \pi^2/4$



Band diagram: $k = K = \pi^2/4$



Band diagram: $k = K = \pi^2/4$

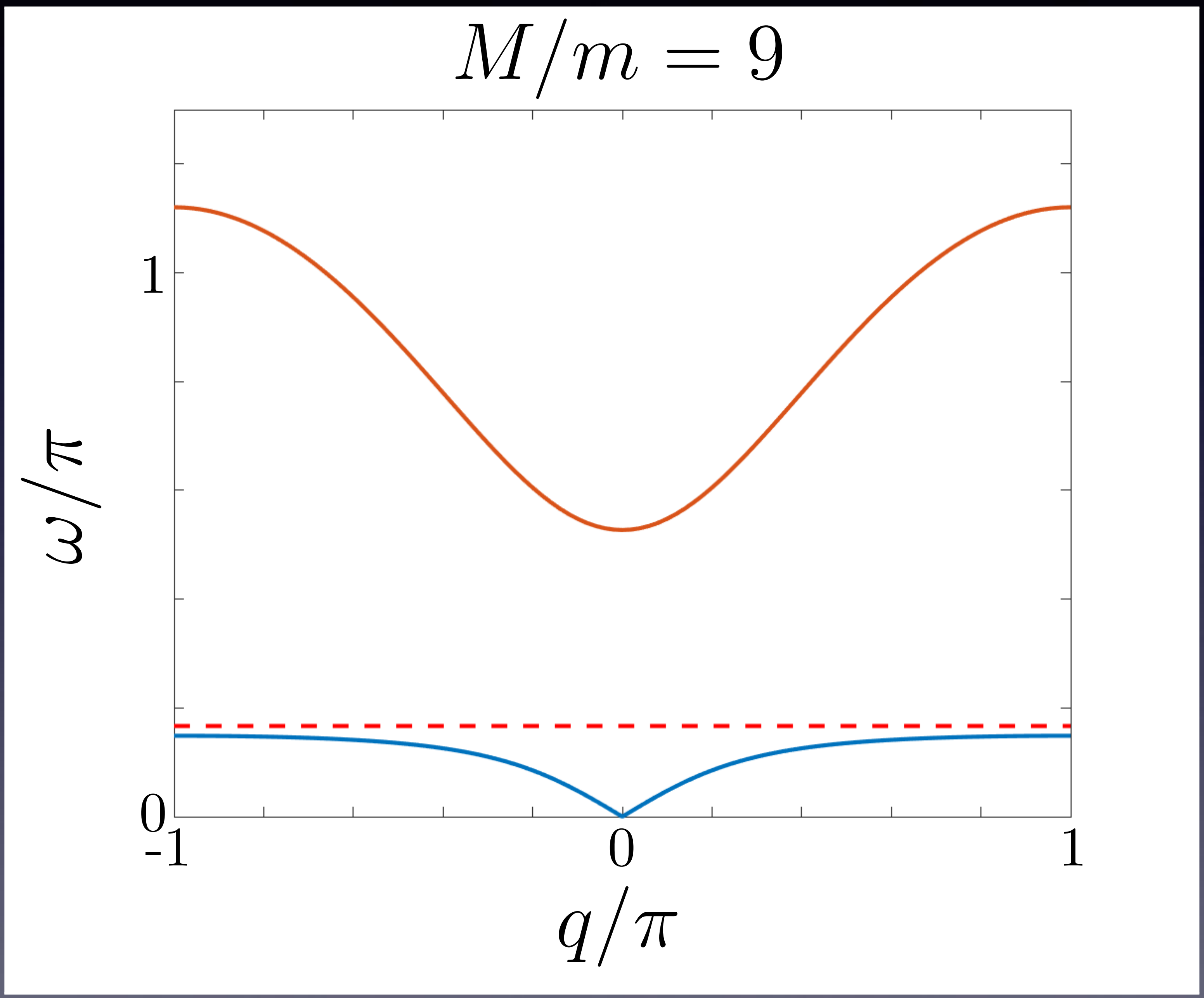


Lower resonance

=

Lower bandgap

Band diagram: $k = K = \pi^2/4$



Lower resonance

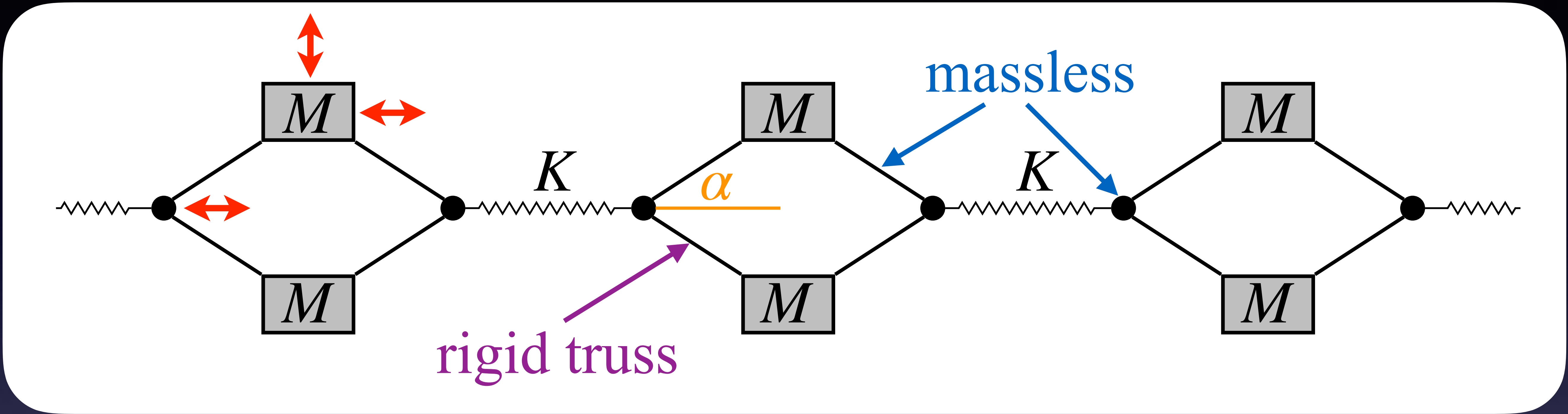
=

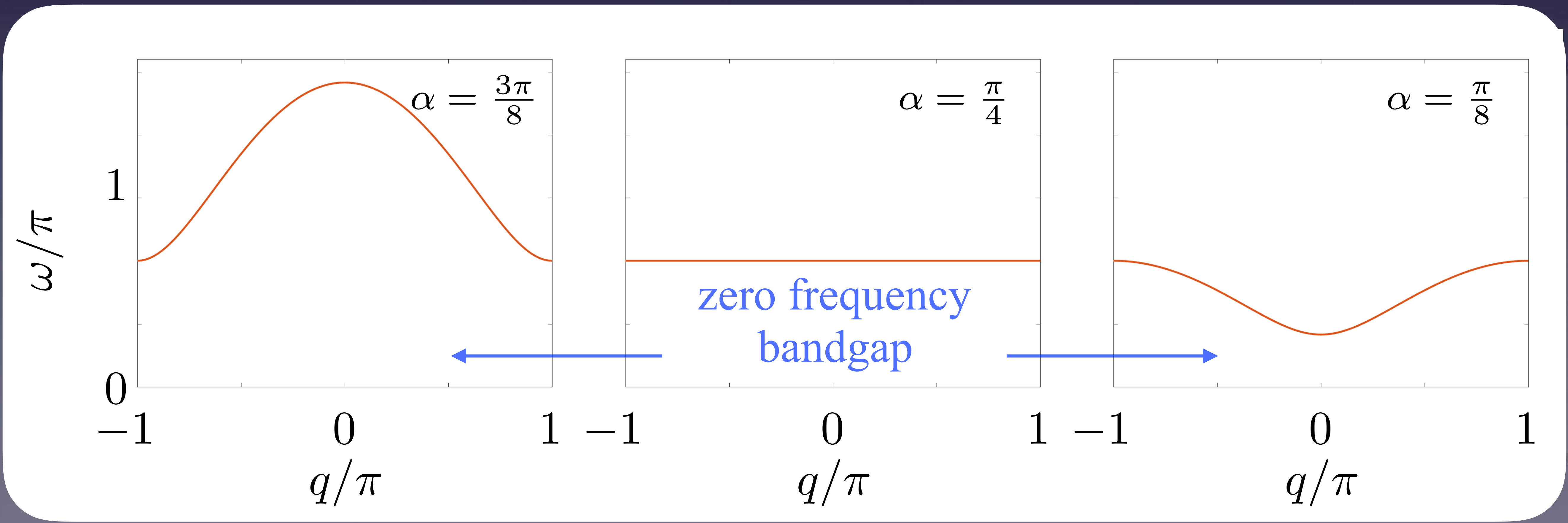
Lower bandgap

But at expense of increased mass

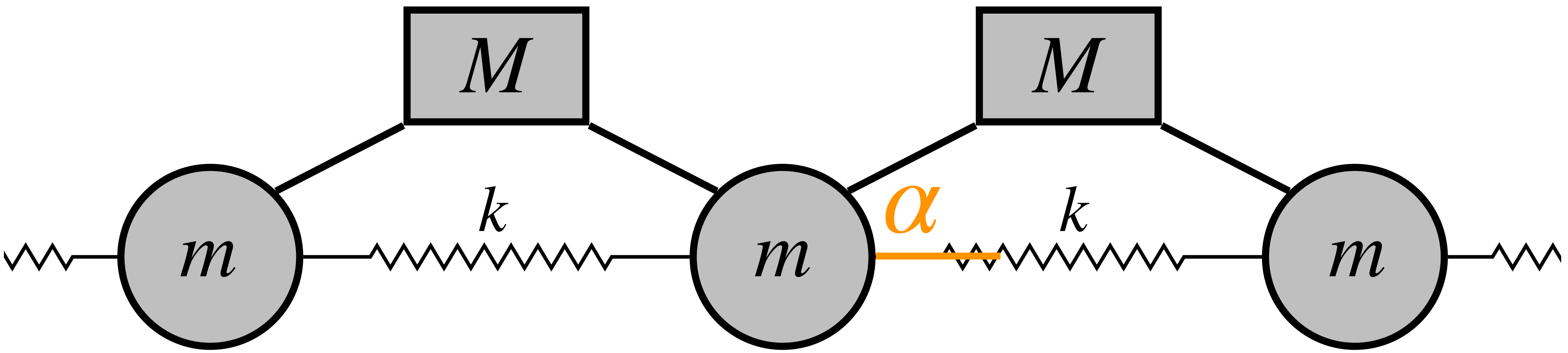
Bobrovnitskii (Acoust Phys, 2014)*

*Milton & Seppecher (2012)

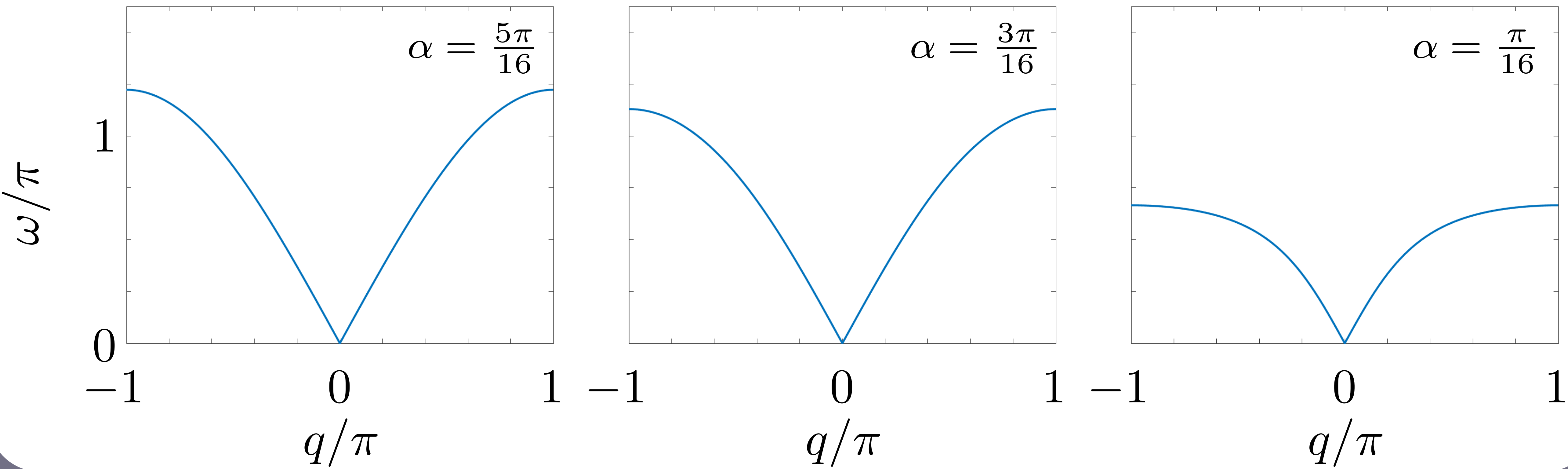




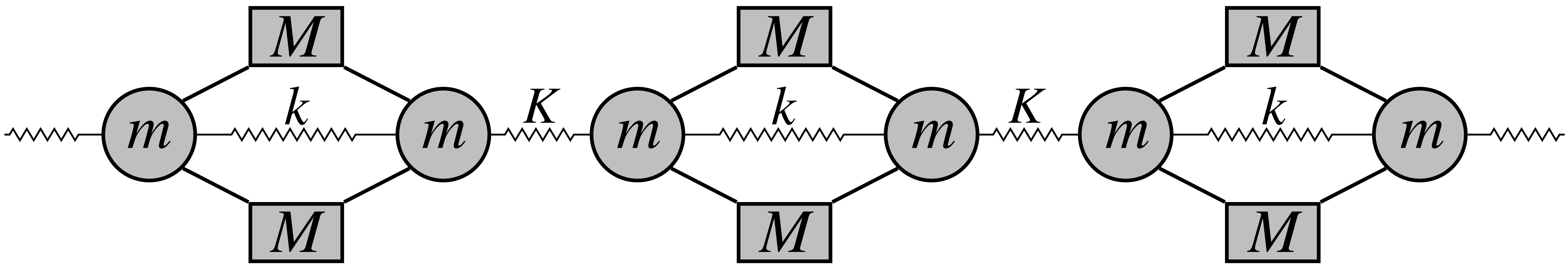
Yilamz & Hulbert (Phys Lett A, 2010)



$M/m = 0.1$

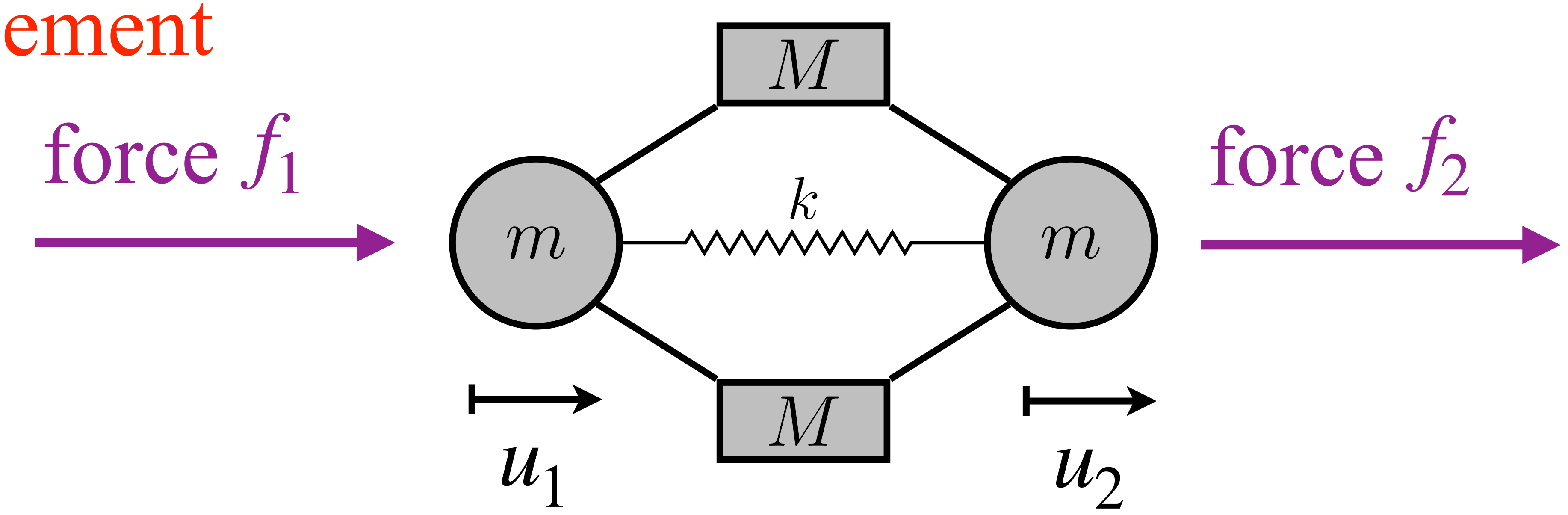


Hybrid chain



Hybrid chain

Basic element



Eqns of Motion (assuming $e^{i\omega t}$)

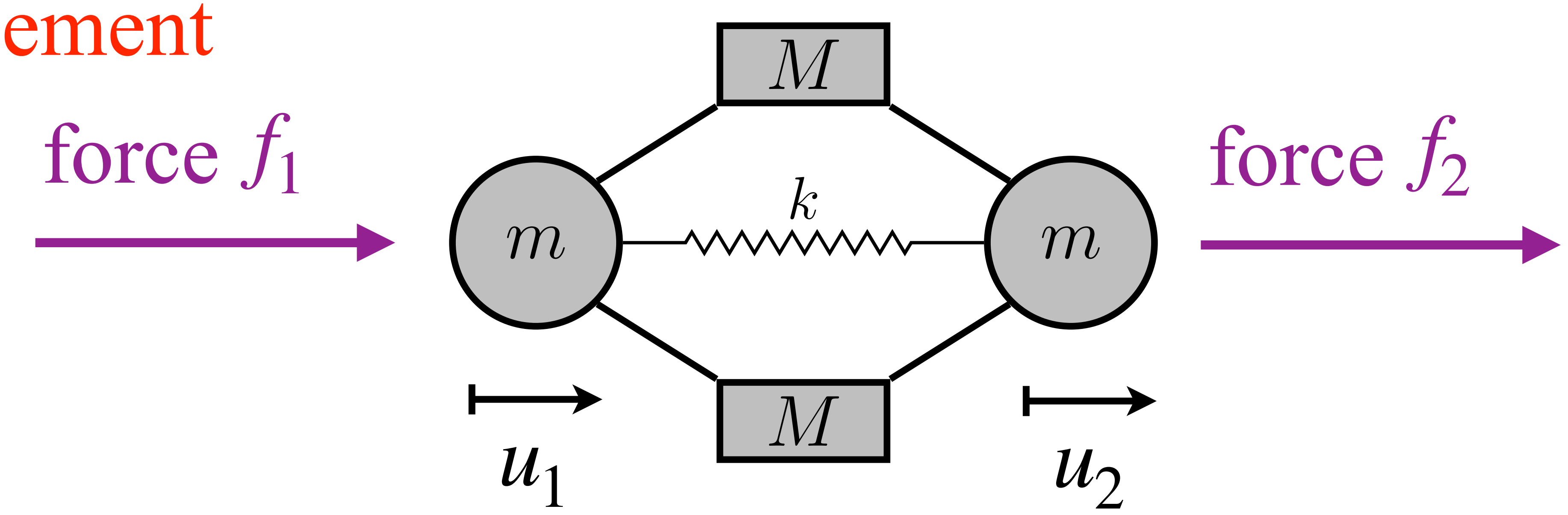
$$-f_1 = \omega^2 \left(m + \frac{M}{2}(1+\gamma^2) \right) u_1 + \frac{\omega^2 M}{2}(1-\gamma^2)\, u_2 + k\,(u_2 - u_1)$$

$$-f_2 = \frac{\omega^2 M}{2}(1-\gamma^2) u_1 + \omega^2 \left(m + \frac{M}{2}(1+\gamma^2) \right) u_2 + k\,(u_1 - u_2)$$

with geometrical parameter $\gamma = \cot \alpha$

Hybrid chain

Basic element

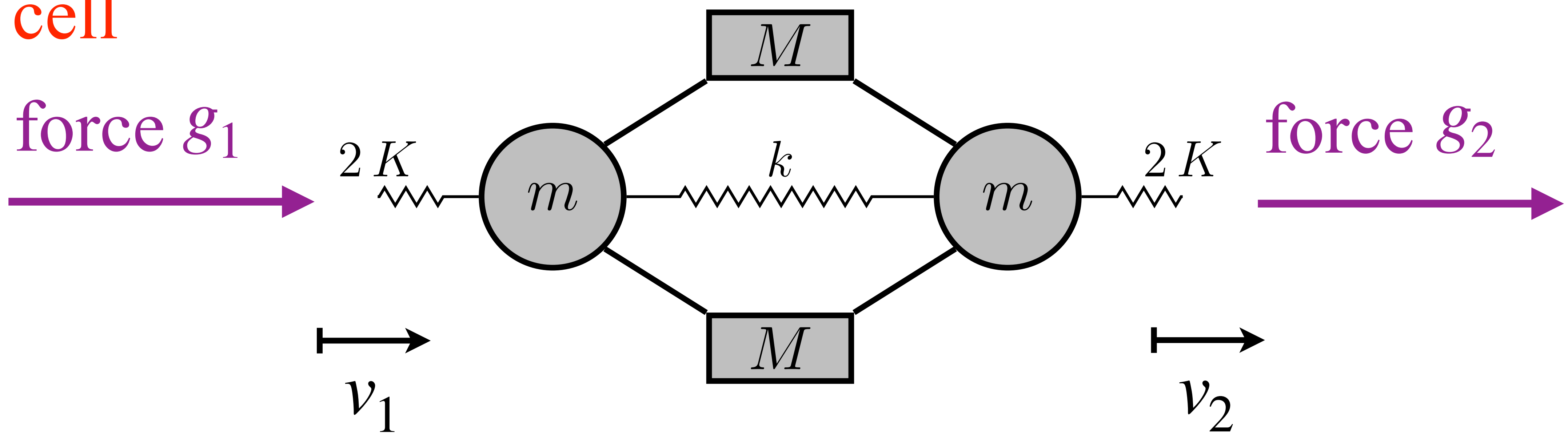


Compliance and stiffness matrices

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = C_0 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = D_0 \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad \text{where} \quad D_0 = C_0^{-1}$$

Hybrid chain

Unit cell

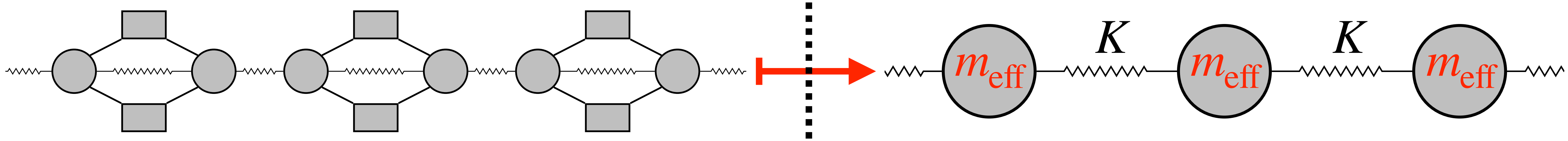


Compliance and stiffness matrices

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = C_0 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = D_0 \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad \text{where} \quad D_0 = C_0^{-1}$$

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = D \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}}_{D = D_0 + \frac{1}{2K} I} \quad \Rightarrow \quad \underbrace{\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = C \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{C = D^{-1}}; \quad \underbrace{C = \begin{bmatrix} c_1 & c_2 \\ c_2 & c_1 \end{bmatrix}}_{\text{symmetry}}$$

“Effective” hybrid chain



$$g_2 = -\,g_1\,e^{iq} \quad \& \quad v_2 = v_1\,e^{iq}$$

$$\Rightarrow \quad c_1 + c_2 \cos q = 0$$

$$m_{\rm eff}\,\omega^2 = 2\,K\,(1 - \cos q)$$

Equating

unit mass

spring

$$m_{\rm eff} = \frac{2\,K}{\omega^2}\left(\frac{c_1 + c_2}{c_1}\right)$$ general

$$= \frac{2\,(m + M)\,\omega_*^2\,\{(1 + K/k)\,\omega_0^2 - \omega^2\}}{\omega_0^2\,(\omega_*^2 - \omega^2)}$$ specific

“Effective” hybrid chain

$$m_{\text{eff}} = \frac{2\,(m+M)\,\omega_*^2\,\{(1+K/k)\,\omega_0^2 - \omega^2\}}{\omega_0^2\,(\omega_*^2 - \omega^2)}$$

$$\omega_*^2 = \frac{2\,k}{M\,(\gamma^2 - 1)}$$

anti-resonance

if $\gamma > 1$

i.e. $\alpha < \pi/4$

$$\omega_0^2 = \frac{2\,k}{m + M\,\gamma^2}$$

out-of-phase

band edge

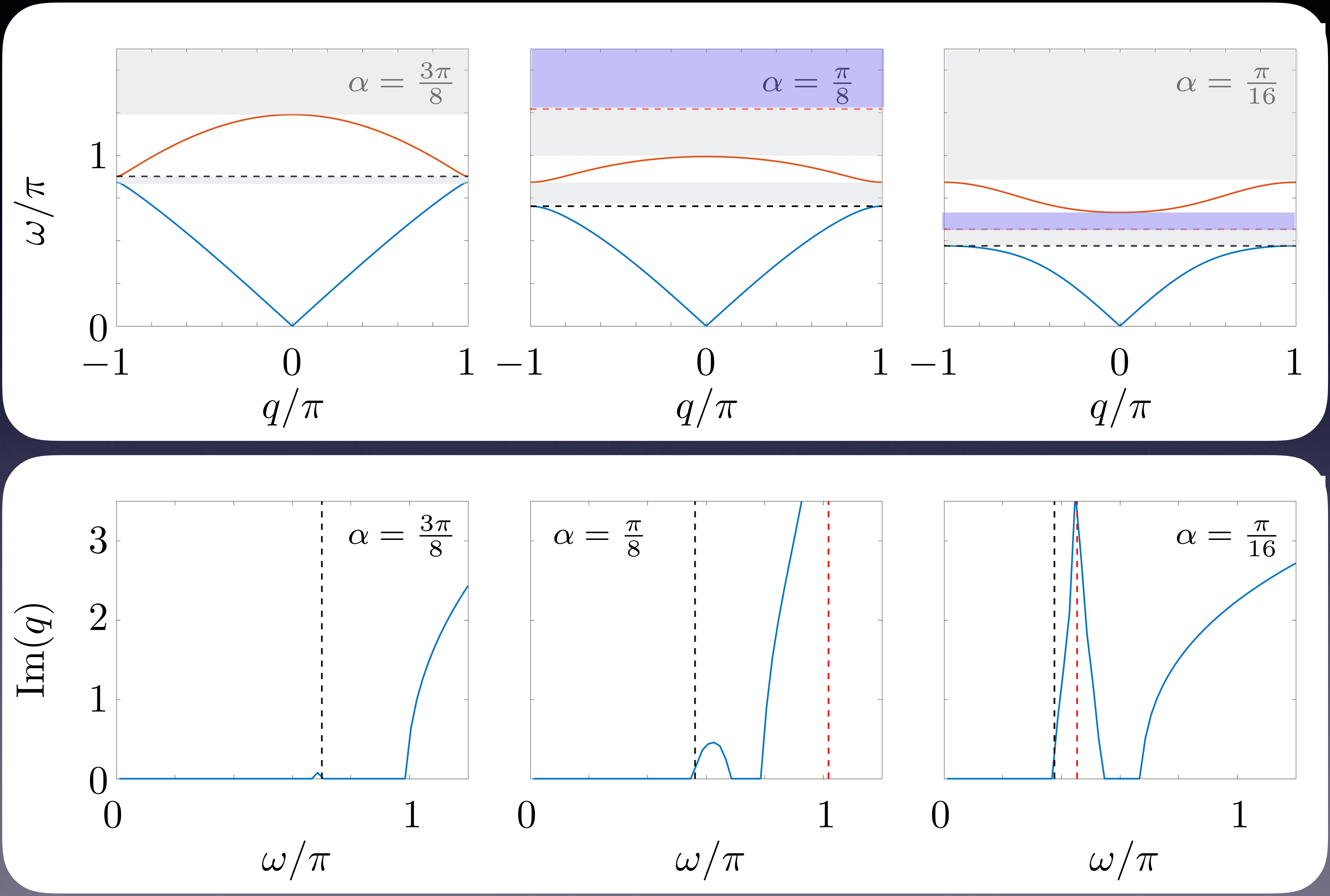
$c_2 = d_2 = 0$

no propagation

$c_1 = c_2$

$\det C_0 = 0$

Band diagrams: $k = K = \pi^2/4; M = m/10$

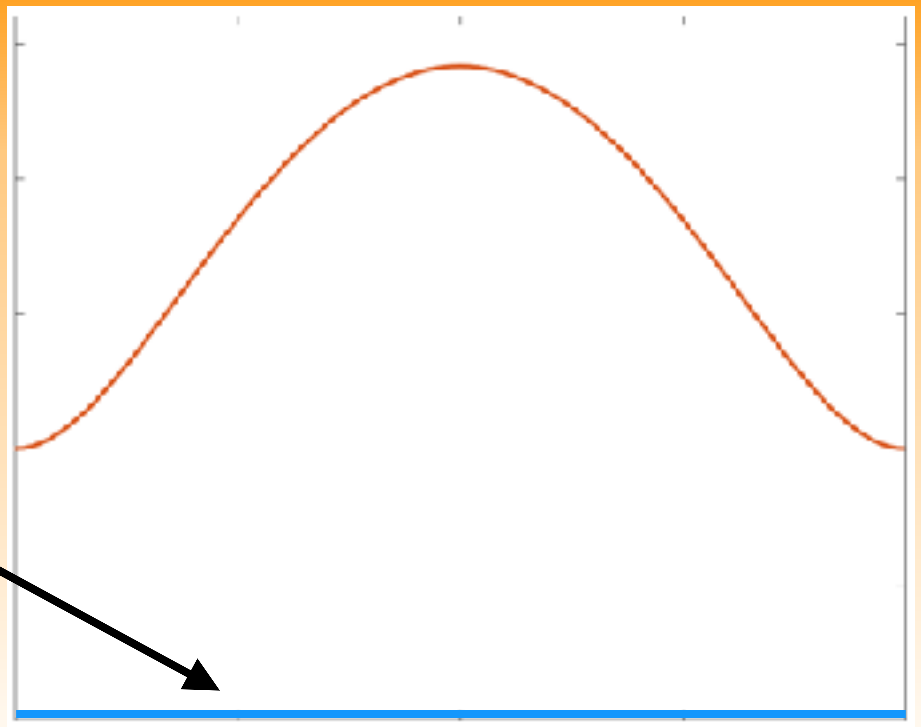


Hybrid chain: limits

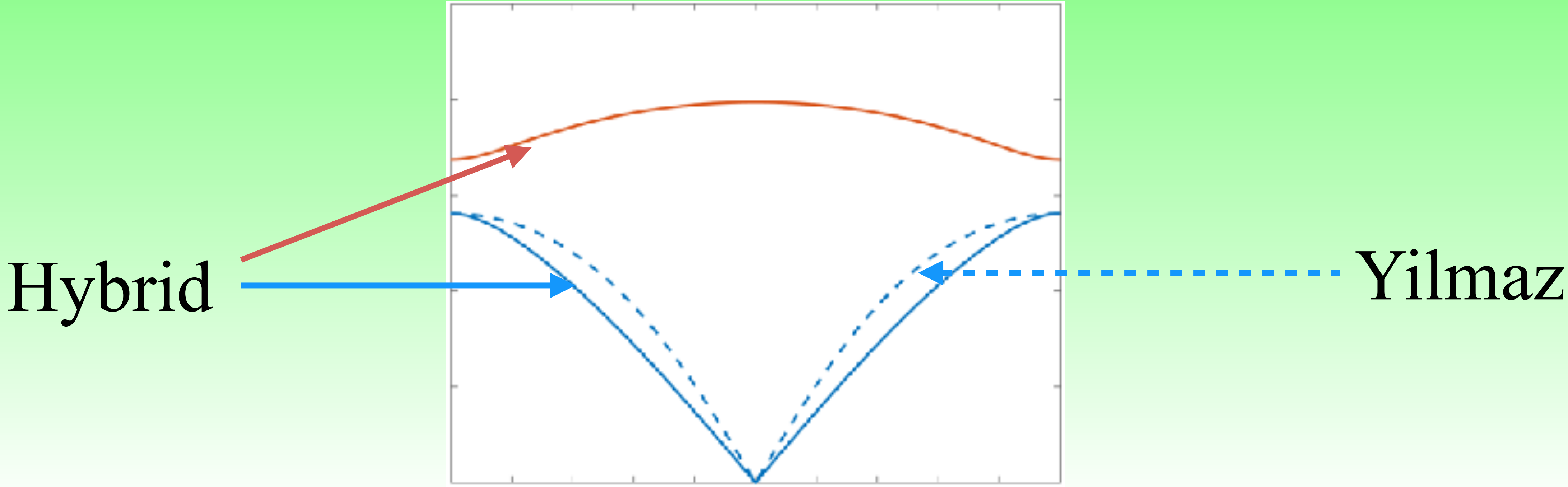
Bob′skii limit: $m, k \to 0$

$$\Rightarrow \ \omega_*, \omega_0 \to 0$$

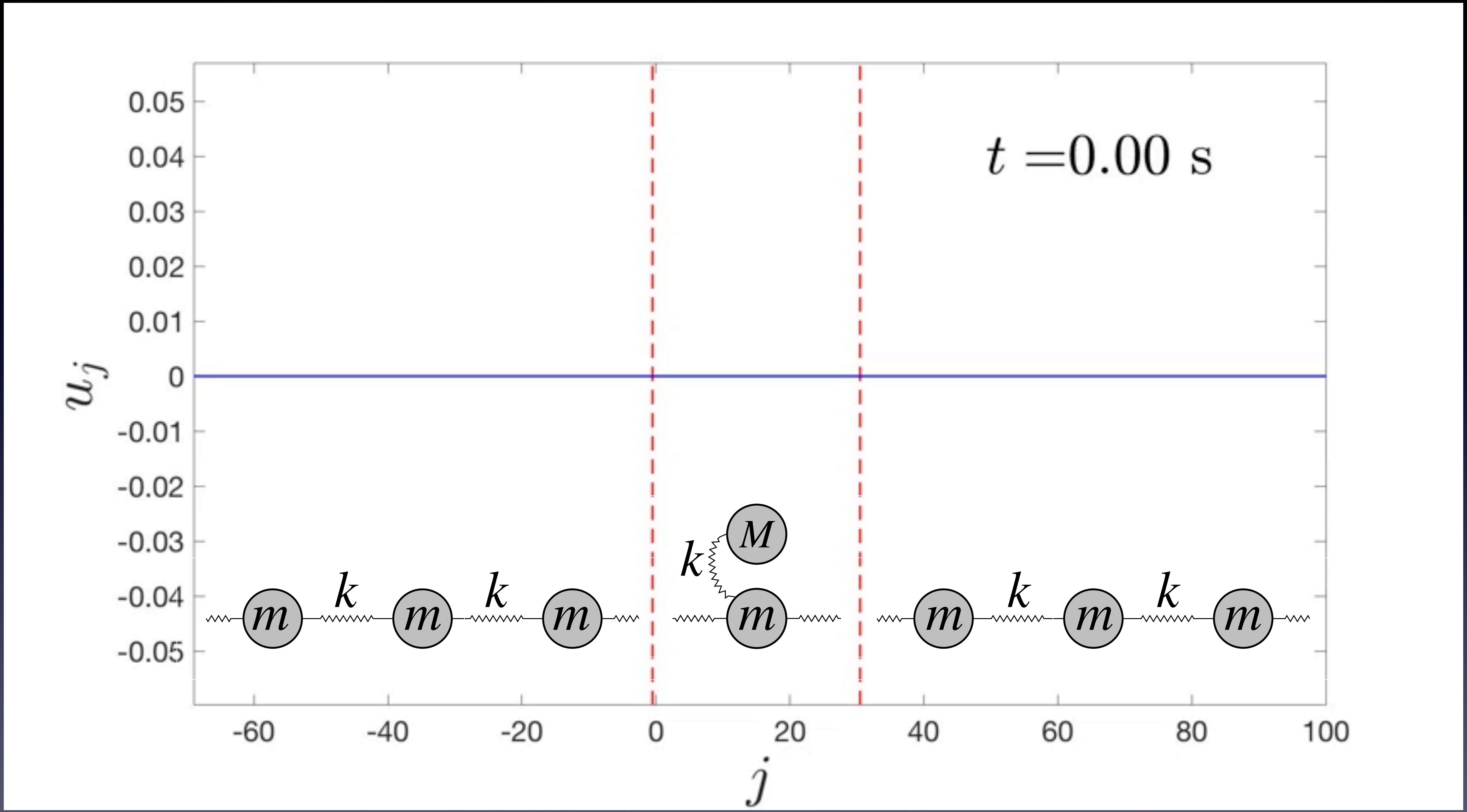
Pushes acoustic branch to $\omega(q) = 0$.



Yilmaz limit: $K \to \infty;\ m \to m/2;$ and $M \to M/2$

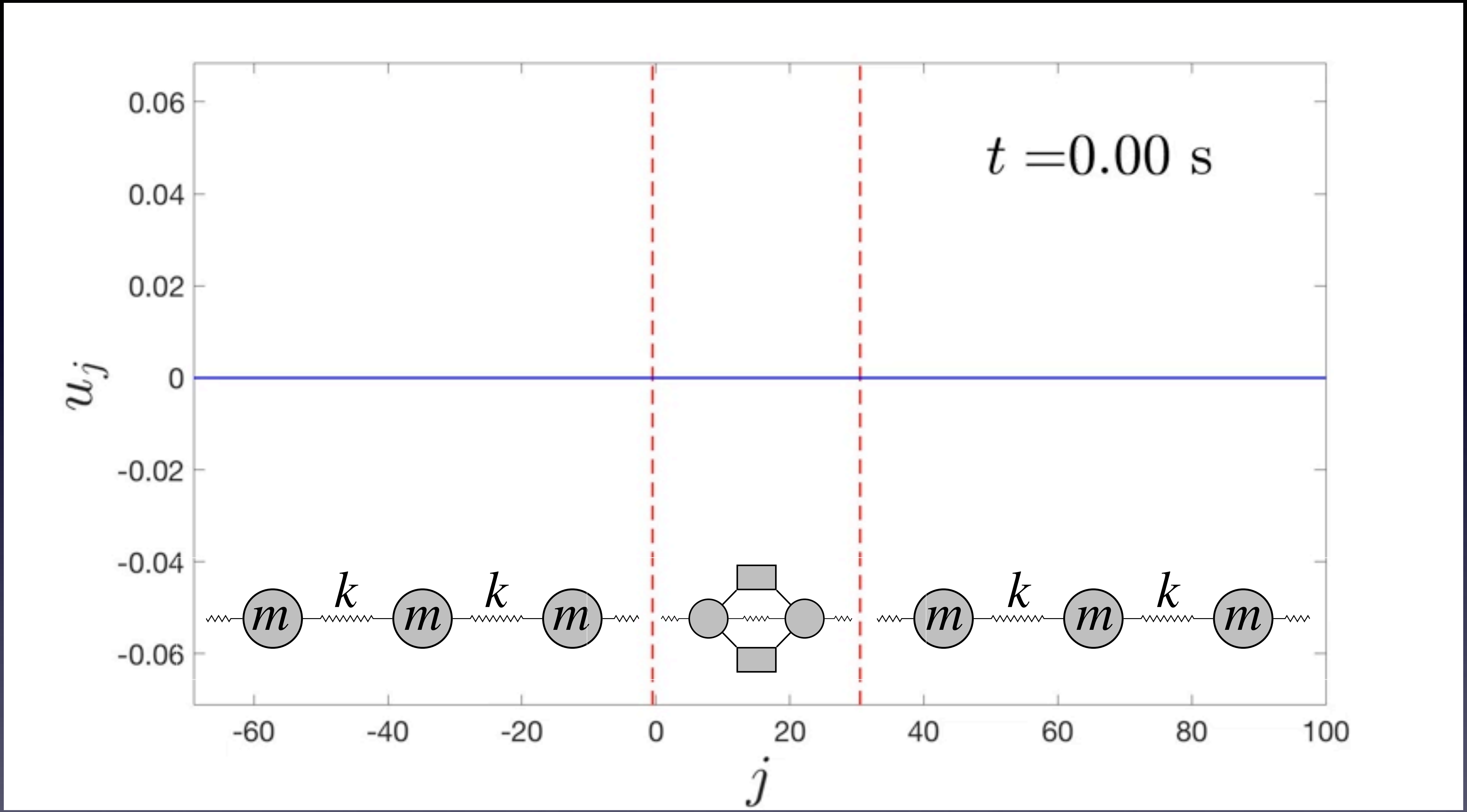


Transmission: Mass-in-mass chain



- 30-unit AMM embedded in monatomic chain
- AMM has fixed added mass $3\,m \Rightarrow M/m = 0.1$
- Incident wave packet with central frequency $\omega/\pi = 0.45$

Transmission: Hybrid chain



- 30-unit AMM embedded in monatomic chain
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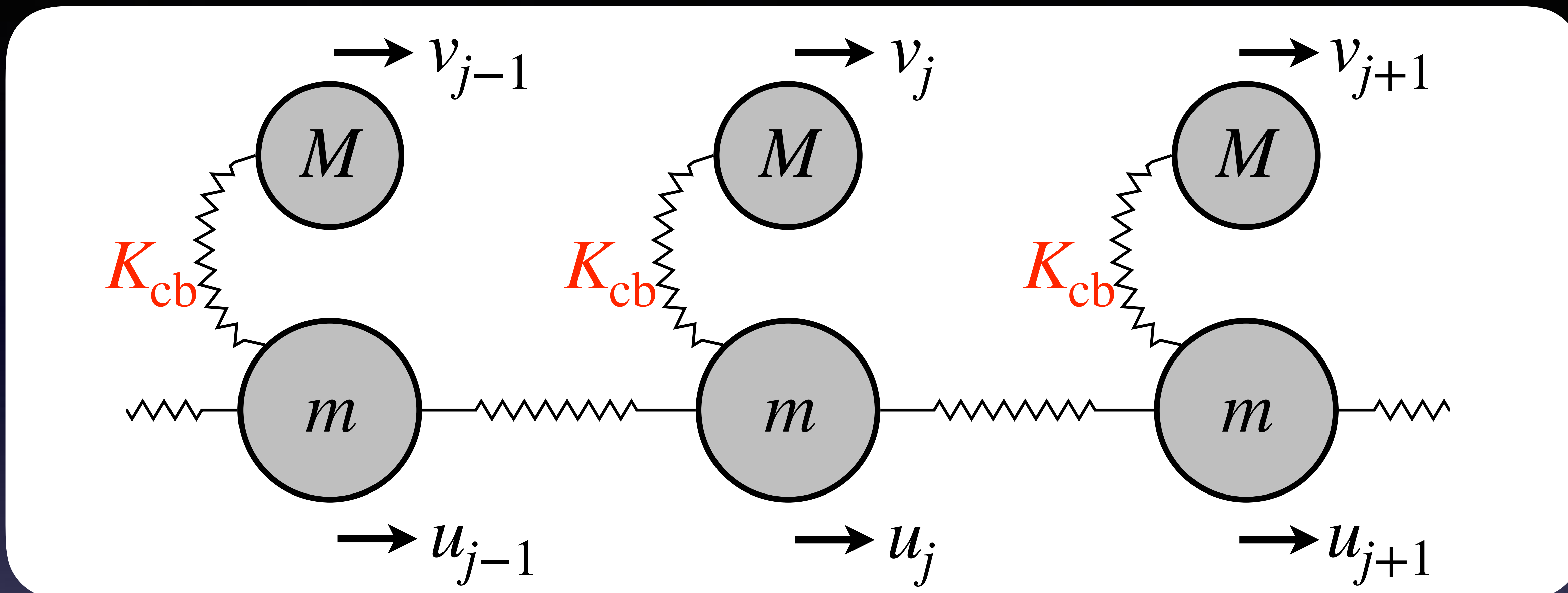
Cubically nonlinear mass-in-mass chain



Equations of motion

$$m\,\ddot{u}_j + k\,(2\,u_j - u_{j-1} - u_{j+1}) + K_1(u_j - v_j) + K_3(u_j - v_j)^3 = 0$$

$$M\,\ddot{v}_j + K_1\,(v_j - u_j) + K_3\,(v_j - u_j)^3 = 0$$

Long-wavelength limit: $q \to 0$

$$\ddot{u} - c_0^2\, u'' + \omega_*^2\, \mathscr{M}\,(u - v) + \Gamma(u - v)^3 = 0$$

$$\ddot{v} + \omega_*^2\,(v - u) + \Gamma\,\mathscr{M}\,(v - u)^3 = 0$$

where $c_0 = \sqrt{k/m}$ = linear wave speed

$\omega_* = \sqrt{K_1/M}$ = anti-resonant frequency

$\mathscr{M} = m/M$

$\Gamma = K_3/m$ = nonlinearity parameter

Weakly nonlinear quasi-monochromatic waves
+ multi-scale analysis

$$u(x,t) = \mu\, A_0(X_1, X_2, T)\, \mathrm{e}^{\mathrm{i}(q\,x - \omega\,t)} + \ldots, \quad \text{etc.}$$

where $X_i = \mu^i\, x$, $T = \mu\, t$, $\mu \ll 1$

Long-wavelength limit: $q \to 0$

$$u(x,t) = \mu\, A_0(X_1, X_2, T)\, \mathrm{e}^{\mathrm{i}(q\,x - \omega\, t)} + \ldots \qquad (X_i = \mu^i\, x,\ T = \mu\, t)$$

Nonlinear-Schrodinger Eqn

$$A_{X_2} + \mathrm{i}\,\alpha\, A_{\xi\,\xi} + \mathrm{i}\,\beta\, A\,|A|^2 = 0$$

for $A(\xi, X_2) \equiv A_0(X_1, X_2, T)$ with $\xi = X_1 - c_{\mathrm{g}}\, T$

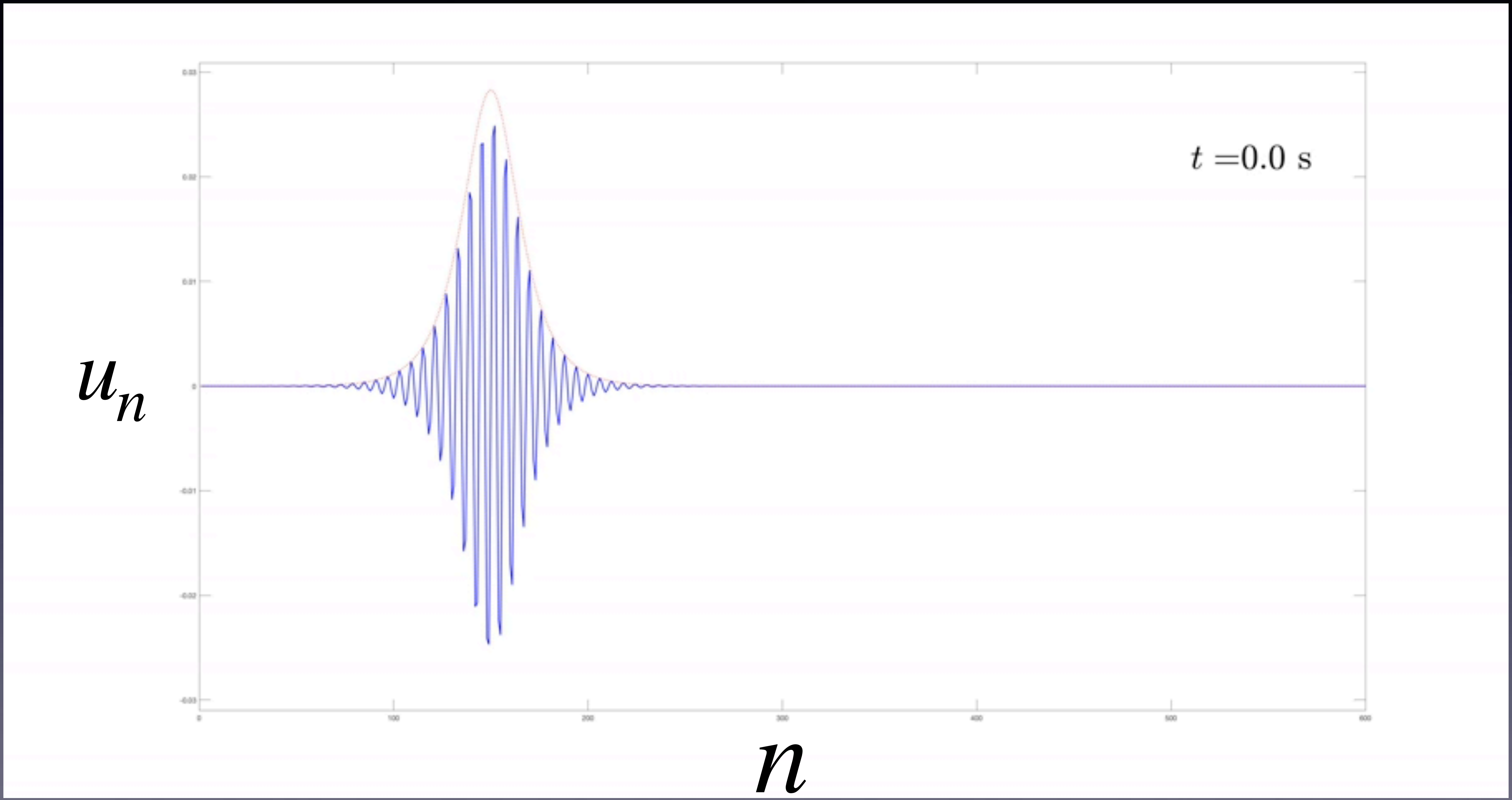
$$c_{\mathrm{g}} = \frac{c_0^2\, q\,(\omega^2 - \omega_*^2)^2}{\omega\,\{(\omega^2 - \omega_*^2)^2 + \omega_*^4\, \mathscr{M}\}}$$

group velocity

$$\alpha = -\,\frac{(c_0^2 - c_{\mathrm{g}}^2)\,(\omega^2 - \omega_*^2)^3 + \mathscr{M}\, c_{\mathrm{g}}^2\, \omega_*^4\,(3\,\omega^2 + \omega_*^2)}{2\, c_0^2\, q\,(\omega^2 - \omega_*^2)^3}$$

$$\beta = \frac{3\,\Gamma\,\omega^6\,\{\omega^2 - \omega_*^2(1 - \mathscr{M})\}}{2\, c_0^2\, q\,(\omega^2 - \omega_*^2)^4}$$

Envelope soliton on acoustical branch



Summary

- AMM chains with lightweight attachments
- Effective mass controlled by geometry
- Generates low-frequency bandgaps
  - + anti-resonance generates deep bandgaps

Future: Nonlinear chains

- Mass-in-mass
  - Optical branch, modulational instability, etc
  - Branch coupling???
- Repeat for Bob'skii, etc
  - Noting control over group velocity sign