Acoustic metamaterial chains involving inertial amplification: Linear case

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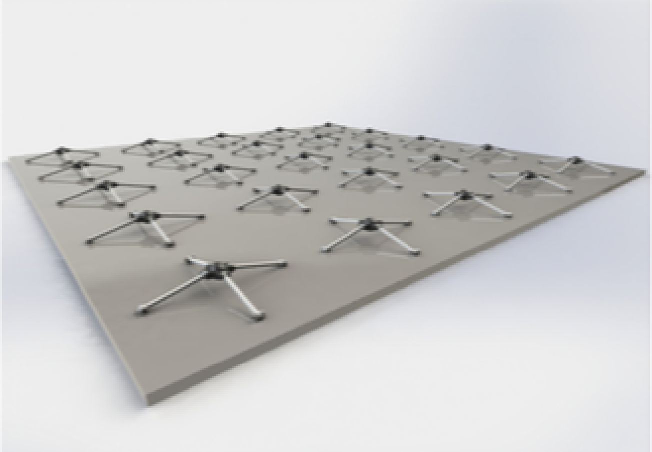
SUB CRUCE LUMEN

Low-frequency noise

Problem

- Low-frequency noise travels farthest.
- Most difficult to isolate (or damp).
- AMMs are designed to isolate low-frequency noise. ✓
- Typically require heavy elements. ✗

AMMs using lightweight attachments



Frandsen et al, J Appl Phys, 2016

- Idea from J-dampers in F1 racing car suspension systems.

AMM as 1D chain

Metamaterials — according to Wikipedia

"A material engineered to have a property that is not found in nature"

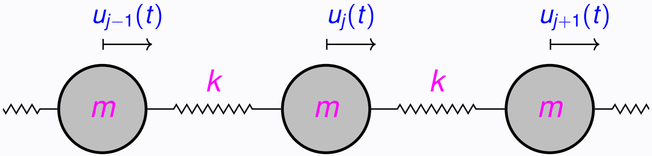
1D chain

where something *"meta"* happens in the boxes.

Simple starting point for analysis

Monatomic chain: Not an AMM

Infinite chain with parameters m = mass and k = spring



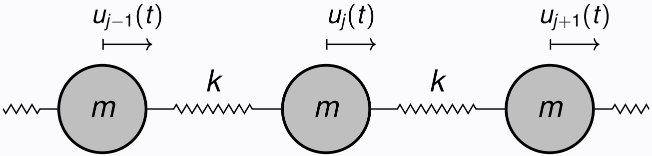
Equations of motion

$$m\,\ddot{u}_j + k\,(2\,u_j - u_{j-1} - u_{j+1}) = 0 \quad \text{for} \quad j \in \mathbb{Z}$$

Simple starting point for analysis

Monatomic chain: Not an AMM

Infinite chain with parameters m = mass and k = spring



Equations of motion

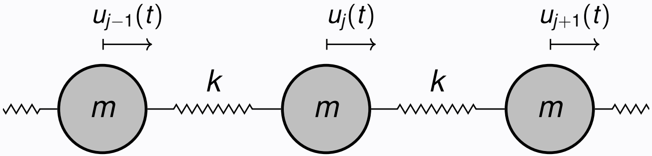
$$-\omega^2\, m\, \widehat{u}_j + k\,(2\,\widehat{u}_j - \widehat{u}_{j-1} - \widehat{u}_{j+1}) = 0$$

- Fourier transform, i.e. write $u_j(t) = \widehat{u}_j \exp(-\mathrm{i}\,\omega\, t)$.

Simple starting point for analysis

Monatomic chain: Not an AMM

Infinite chain with parameters $m =$ mass and $k =$ spring



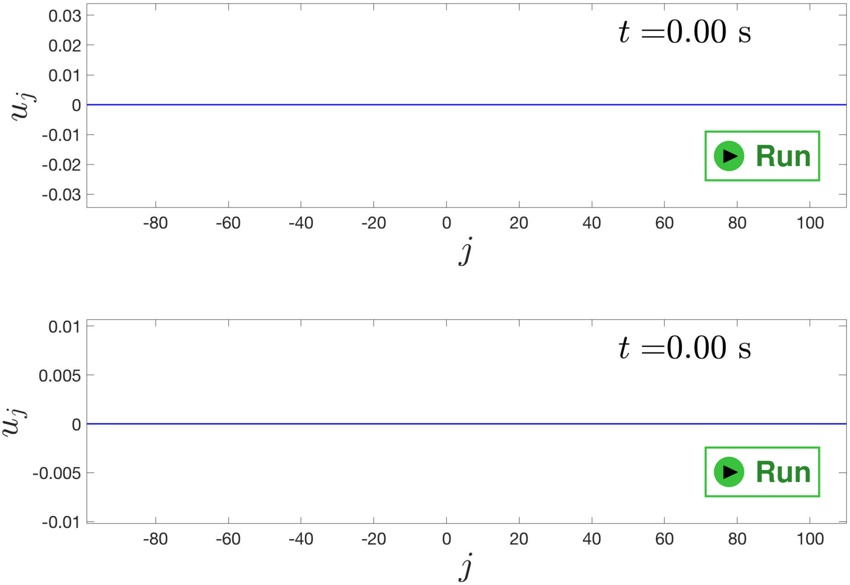
Dispersion relation

$$m\,\omega^2 = 2\,k\;(1 - \cos q)$$

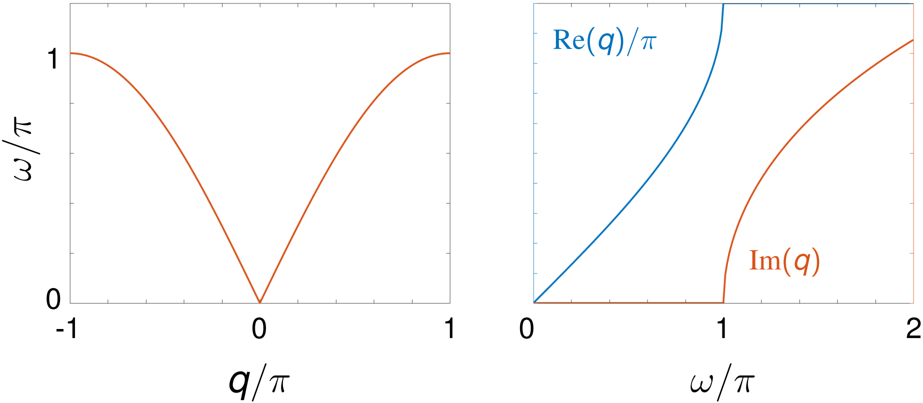
- Fourier transform, i.e. write $u_j(t) = \widehat{u}_j \exp(-\mathrm{i}\,\omega\,t)$.
- Apply Bloch–Floquet condition (quasi-periodicity)

$$\widehat{u}_j = U \exp(\mathrm{i}\,q\,j) \quad \text{where} \quad q = \text{wavenumber} \in (-\pi.\pi].$$

$m = 1$; $k = \pi^2/4$; $\omega/\pi = 0.6$ (top), 1.2 (bottom)

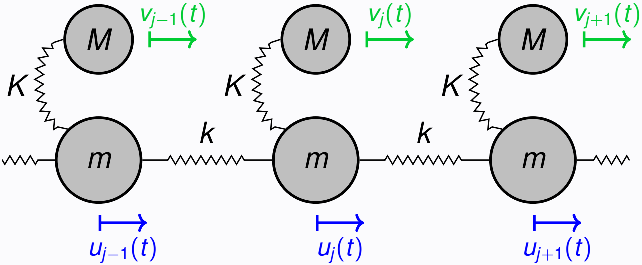


Band diagrams: $m = 1$; $k = \pi^2/4$



Turn it into a metamaterial

Standard AMM: Mass-in-mass chain



Equations of motion

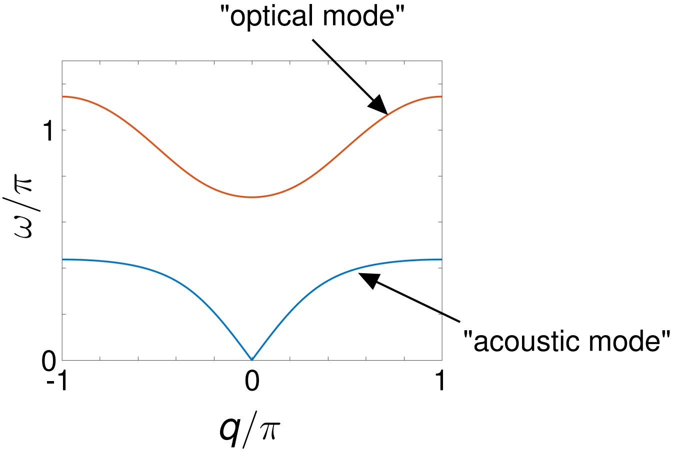
$$m\,\ddot{u}_j + k\,(2\,u_j - u_{j-1} - u_{j+1}) + K(u_j - v_j) = 0$$

$$M\,\ddot{v}_j + K\,(v_j - u_j) = 0$$

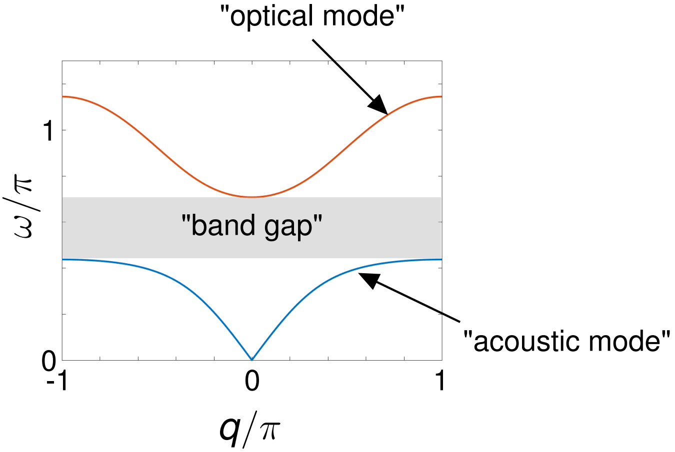
Dispersion relation

$$m\,M\,\omega^4 - \{K\,(m + M) + 2\,k\,(1 - \cos q)\}\,\omega^2 + 2\,(1 - \cos q) = 0$$

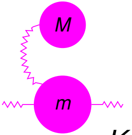
Band diagrams: $M = m = 1$; $K = k = \pi^2/4$

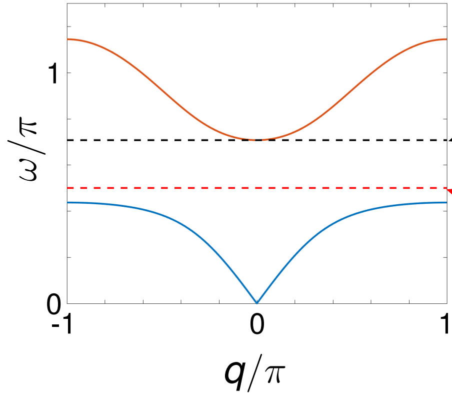


Band diagrams: $M = m = 1$; $K = k = \pi^2/4$



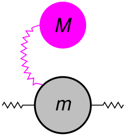
Band diagrams: $M = m = 1;\ K = k = \pi^2/4$



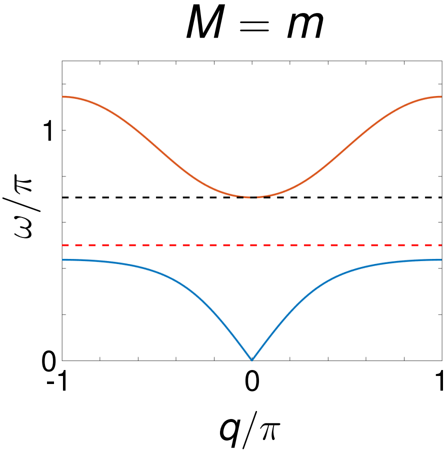


resonance: $\omega^2 = \omega_0^2 \equiv \dfrac{K\,(m + M)}{m\,M}$

anti-resonance: $\omega^2 = \omega_*^2 \equiv \dfrac{K}{M}$

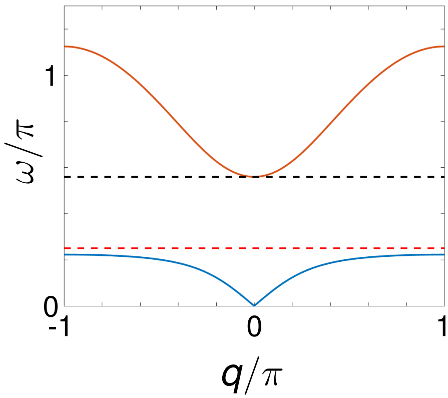


Band diagrams: $m = 1$; $K = k = \pi^2/4$



Band diagrams: $m = 1$; $K = k = \pi^2/4$

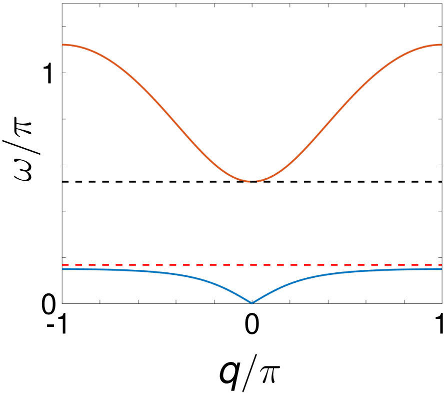
$M = 4m$



Lower resonances
= lower band gap

Band diagrams: $m = 1$; $K = k = \pi^2/4$

$M = 9m$



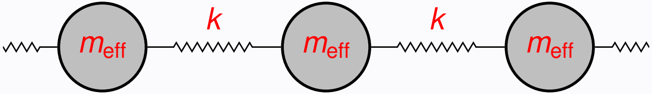
Lower resonances
= lower band gap

But mass increases

Why is mass-in-mass chain an AMM?

"...a property that is not found in nature"

Mass-in-mass chain $\mapsto$ "effective" monatomic chain



with dispersion relation (from earlier)

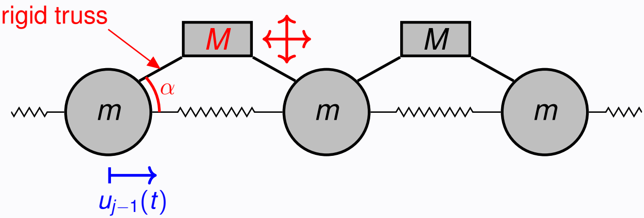
$$\frac{m_{\text{eff}}\,\omega^2}{k} = 2\,(1-\cos q) \in [0,4] \quad \text{for} \quad q \in \mathbb{R}$$

where

$$m_{\text{eff}} = m + \frac{M\,\omega_*^2}{\omega_*^2 - \omega^2} \; : \quad |\omega_*| \gg 1 \quad \text{for} \quad \omega \sim \omega_*$$

AMM with lightweight attachments (+ geometry)

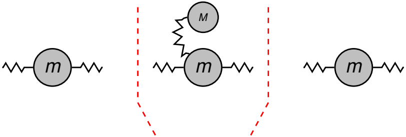
Simplest variant (Yilmaz & Hulbert, Phys. Lett. A, 2010)

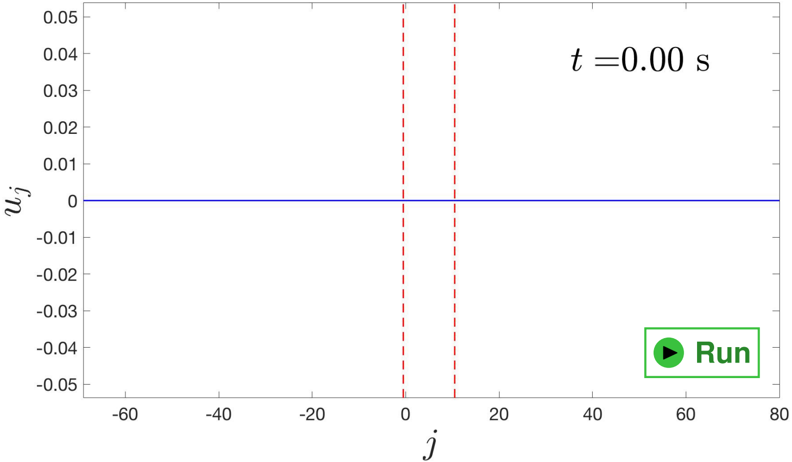


Equation of motion

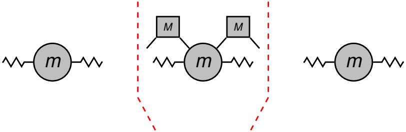
$$\left(m + \frac{1}{4} M \left(1 + \cot^2 \alpha\right)\right) \ddot{u}_j + \frac{1}{4} M \left(1 - \cot^2 \alpha\right) \left(\ddot{u}_{j-1} + \ddot{u}_{j+1}\right)$$
$$+ k \left(2 u_j - u_{j-1} - u_{j+1}\right) = 0$$

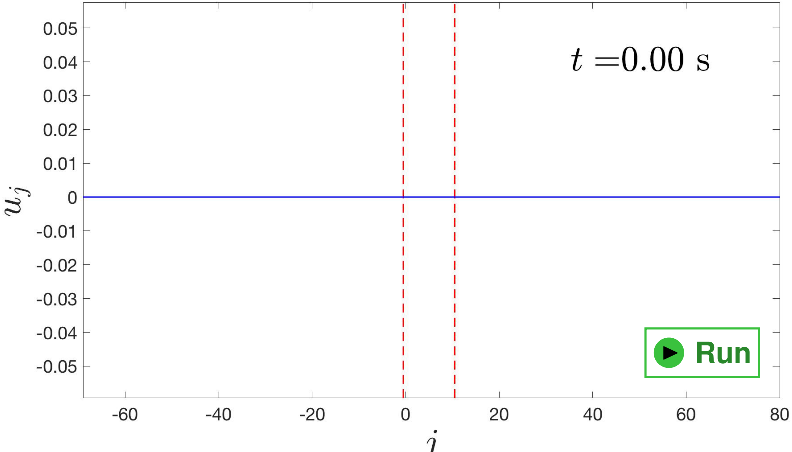
Mass-in-mass: $M = m/10$; $\omega = 6\pi/10$





Yilmaz & Hulbert: $M = m/9$; $\omega/\pi = 0.6$; $\alpha = \pi/16$





Final slide

Summary

- Attached masses excite resonance/anti-resonance to suppress low-frequency vibrations.
- Lightweight attachments can achieve this using geometry.

Next

- Nonlinearity (physical or geometrical) to transfer energy to higher frequencies (and damp).
- Modulational instability between acoustical and optical branches for efficient energy transfer.