An Experimental Model of Wave-Induced Motions of an Ice Floe

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Abstract
Approximately ten per cent of the ocean surface is frozen into a layer of sea ice. Ocean surface waves penetrate deep into the ice-covered ocean. Waves breakup the ice, cause ice floes to raft and contribute to formation of new ice. They hence play a key role in extent and strength of the ice cover. An experimental model of wave-ice interactions was implemented using the wave basin facility at Plymouth University. A single floe was subjected to monochromatic waves, using different amplitudes and frequencies. Two different synthetic materials were used to model the ice. Only a loose mooring restricted the floe. The elastic plate motion was measured using a non-intrusive motion tracking system and the depth of fluid on the plate surface was measured simultaneously. Preliminary results in which the experimental measurements are compared to a two dimensional thin elastic plate model are presented.

Introduction
The Marginal Ice Zone is a region of broken ice which forms at the boundary of the open and frozen ocean. It is often subject to intense wave activity and waves are believed to play a critical role in determining its extent and morphology. Understanding the complex interaction between waves and ice requires comprehending a number of important processes, including both floe breaking and wave attenuation. However, the most critical to determine is the attenuation of waves, because it is impossible to estimate wave induced floe breaking without knowing the wave intensity. The successful modeling of wave propagation in the MIZ has applications to wave forecasting [2], offshore engineering, and climate modeling.

Recently, an experimental measurement using disposable wave sensors was conducted in the Antarctic and comparison was made with the observed trends in sea ice extent [3]. This study showed that ocean was play a much greater role in controlling ice extent than was previously thought. Further analysis of this data appeared in [6] which showed that the attenuation varies with inverse square of wave period. At present, no theory exists which explains this dependence.

The experimental study of elastic bodies in a wave flume began with researchers trying to understand the motion of very large floating structures, motivated by proposals to build floating airports [12]. In the context of wave-ice interactions a series of experiments were performed on circular floating elastic plates by [9, 8]. The present work can be thought of as an extension of this study. In the work of [9, 8], two key unphysical constraints were applied to the floating plate. The plate was moored so that all surge motion was restricted to zero and a barrier was built around the elastic plate to insure no fluid could entered onto the plate surface (i.e. that the overwash was zero). In our present study, the plate is only loosely moored and no barrier is placed around the plate. Furthermore, we consider here rectangular plates, which allow comparison between the two and three dimensional hydroelastic theories.

The standard model for ice floes is to assume they can be modelled as a rotating plate with thickness. The experiments of [9, 8] showed that this model performed well (with their inclusion of a barrier and surge restricting mooring). The elastic plate model is an extension of the standard linear potential flow model for rigid bodies to include elastic modes [1].

Experimental Setup
The laboratory facility consists of a directional wave basin of width of 10 m, length 15.5 m and water depth 0.5 m (see Fig. 1). The tank is equipped with twenty individually controlled active piston wave makers, which are capable of absorbing incoming waves by measuring the force on the paddle and controlling the velocity. At the opposite end, wave energy is dissipated by a beach of slope 1:10. A reflection analysis in the centre of the basin shows that the combined effect of the active pistons and the beach ensure an overall level of reflection lower than 1% of the incoming energy. Remaining reflected energy (normally confined within low frequency components) is removed.

At a distance of 2 m from the wave maker, a plastic sheet was deployed to simulate an ice floe. Two different types of plastic were tested: a polystyrene plastic with density 0.905 g cm⁻³ and Young’s modulus 1600 MPa; and PVC (FOREX®) plastic with density 0.500 g cm⁻³ and Young’s modulus 500 MPa. Note that polypropylene has density similar to sea ice but has different rigidity. PVC has a rigidity comparable to sea ice but substantially lower density, which results in a larger freeboard. Polypropylene sheets were provided with thicknesses 5 mm, 10 mm, 20 mm and 40 mm; PVC was provided with thicknesses 5 mm, 10 mm and 19 mm. The sheets were cut into square floes with side lengths \( L_{plate} = 1 \text{ m} \).

The experimental set-up was designed to represent the full scale wave-ice interactions in scale 1:100.

At the wave maker, waves were generated by imposing three different wave periods, namely \( T = 0.6 \text{ s}, 0.8 \text{ s}, \) and \( 1 \text{ s} \), with corresponding wavelengths \( \lambda_{wave} = 0.56 \text{ m}, 1 \text{ m} \) and \( 1.56 \text{ m} \), respectively. The wave fields therefore tested conditions in which the waves were shorter than, equal to and longer than the floe. The wave amplitude, \( a \), was selected so that the wave steepness \( ka \), where \( k \) is the wavenumber, matched the following values: 0.04, 0.08, 0.1 and 0.15. This range includes gently sloping waves \( ka = 0.04 \) and 0.08) as well as storm-like waves \( ka = 0.1 \) and 0.15), without reaching the breaking limit.

A series of experiments were conducted in which spherical polystyrene markers we placed on the surface of the elastic
Figure 1. Directional wave basin and experimental set-up.

Figure 2. Overwash of a polypropylene floe (right panels) and PVC floe (left panels) for a wave field with wavelength equal to the plate length and plate thickness of 10 mm.

Figure 3. Example of water surface elevation over a polypropylene plastic sheet of thickness 10 mm in a wave field of wavelength 1 m and steepness $ka = 0.15$.

Floating Elastic Plate in Two Dimensions

We present here a simplified model for a floating elastic plate which assumes that the plate can be modelled using thin plate theory and that the draft is negligible. Such a model has become standard as a model for ice floes and a detailed description of applications and theory can be found in [11]. Such a model was the basis of the work of [9, 8]. A key part of the present experimental study is validate this model and to establish its range of applicability. The problem of a two-dimensional floating elastic plate of finite length and zero draft is the simplest and best-studied problem in hydroelasticity, and as such it is the ideal starting point to compare with experiments. Solutions in the frequency domain were first presented by [7, 10]. It is assumed that the plate occupies the region $(-L, L)$ and that the water is of constant depth $h$. The fluid velocity potential is denoted by $\phi$. The problem is to determine the unknown coef-
The equation for the diffraction and radiation potentials is solved using the two-dimensional Green function. The equations for the diffraction and radiation potentials are as follows,

\[ \phi^D(x) = \phi^1(x) + \int_{-L}^{L} G(x, x') \alpha \phi^D(x') dx', \]

\[ \phi^R(x) = \int_{-L}^{L} G(x, x') \left( \alpha \phi^R(x') + i \omega w_v(x') \right) dx'. \]

The Green function satisfies

\[ \Delta G(x, z, x') = 0, -h < z < 0, \]

\[ \partial_z G = 0, z = -h, \]

\[ \partial_z G - \alpha G = \delta(x - x'), z = 0, \]

and the Sommerfeld radiation condition that the waves are outgoing at infinity. The Green function for \( z = 0 \) is given by

\[ G(x, x') = \sum_{p=0}^{\infty} \frac{e^{-ik|x-x'|}}{\pi k_p}, \]

where \( k_p \) are the roots of the dispersion equation

\[ \alpha + k_p \tan(k_p h) = 0, \]

with \( k_0 \) being purely imaginary with negative imaginary part and \( k_p \) for \( p \geq 1 \) being purely real with positive real part ordered with increasing size [4]. It is noted that, since the problem is two dimensional, the transmission coefficient \( T \) can be defined as

\[ T = \lim_{x \to \infty} \frac{\phi(x, 0)}{\phi(x)}, \]

The elements of the real added mass and real damping matrices \( A \) and \( B \) are defined as

\[ \omega^2 A_{\nu \nu} + i \omega B_{\nu \nu} = \omega^2 \rho \int_\Omega \phi^R \phi^R dx, \]

and the elements of the force vector \( f \) are defined as

\[ f_{\mu} = i \omega \rho \int_\Omega \left( \phi^1 + \phi^D \right) \phi^R \phi^R dx. \]

The solution to equations (1) can be written in matrix form as

\[ \left( K + C - \omega^2 M - \omega^2 A(\omega) - i \omega B(\omega) \right) \xi = f(\omega), \]

where the elements of the matrices are given above.

**Results**

We present here preliminary results looking at the comparison of the simple two-dimensional linear model based on potential flow theory and the measured plate motions. The motion was measured on a four by four grid of points. The plastic sheets were placed perpendicular to the incident wave direction. We compare the measured data with our simplified two dimensional model along four lines on the plate. Figure 4 shows the comparison for the 10mm PVC sheet for a period of 0.6s. This test configuration has been chosen because it is one of the test which exhibited the greatest overshoot. The steepness was 0.04. The solid line is the two-dimensional model prediction and the thin lines represent four lines along the plate. Figures 5 and 6 are the equivalent plots except the wave steepness is increased to 0.1 and 0.15 respectively. These figures show that the simplified model gives reasonable prediction even when the steepness

\[ \Delta \phi = 0, -h < z < 0, \]

\[ \partial_z \phi = 0, z = -h, \]

\[ \omega^2 \phi = \partial_z \phi, x \notin (-L, L), z = 0, \]

\[ -i \omega \sum_{v=0}^{\infty} \xi_v w_v = \partial_z \phi, x \in (-L, L), z = 0, \]

\[ \sum_{v=0}^{\infty} \xi_v \left( 1 + \beta^2 \omega^2 \right) w_v - \omega^2 \gamma \sum_{v=0}^{\infty} \xi_v w_v = i \omega \phi, x \in (-L, L), z = 0, \]

where \( w_v \) are the modes of vibration of an elastic plate which satisfy

\[ \partial^2 w_v = \lambda^2 w_v, \]

\[ \left. \partial^2 w_v \right|_{x = -L} = \partial^2 w_v \left|_{x = L} = 0, \right. \]

\[ \left. \partial^2 w_v \right|_{x = -L} = \left. \partial^2 w_v \right|_{x = L} = 0, \]

and are normalized so that

\[ \int_{-L}^{L} \partial \phi \partial \phi, dx = \delta_{\nu \mu}, \]

where \( \delta_{\nu \mu} \) is the Kronecker delta. Note that the appropriate Sommerfeld radiation condition is imposed at infinity. The non-dimensional stiffness is given by \( \beta = D/\rho gl^4 \) where \( D \) is the bending stiffness, \( \rho \) is the fluid density, and \( g \) is the gravitational acceleration. The non-dimensional mass is given by \( \gamma = mH/pl \) where \( m \) is the mass density of the plate and \( H \) is the plate thickness.

The velocity potential is expanded as

\[ \phi = \phi^1 + \phi^D - i \omega \sum_{v=0}^{\infty} \xi_v \phi^R, \]

where

\[ \phi^1 = e^{ikx}, \]

is the incident wave of unit amplitude and \( k \) is the wave number (which is the real positive solution of the dispersion equation \( k \tan(kh) = \omega^2 \)). The diffraction potential \( \phi^D \) is found by solving equation (1) and

\[ -\partial_z \phi = \partial_z \partial \phi, x \in (-L, L), z = 0, \]

The radiation potentials \( \phi^R \) are found by solving equation (1) and

\[ w_v = \partial_z \phi^R, x \in (-L, L), z = 0, \]

We define the stiffness matrix by

\[ K = [\beta^2 \omega^2], \]

where \([\ldots]\) denotes a diagonal matrix and the mass matrix by

\[ M = \gamma I, \]

where \( I \) is the identity matrix. The hydrostatic restoring matrix is defined by

\[ C = I. \]
Figure 4. Motion of the plate (thin lines) for a 10mm PVC sheet for a period of 0.6s. The steepness was 0.04. The thicker line is the two-dimensional model prediction.

Figure 5. As in Figure 4 except the steepness was 0.1.

Figure 6. As in Figure 4 except the steepness was 0.15.

is increased and there is significant overwash. The phase of the wave is given by $\theta$ and $\eta$ is the displacement of the plate.

Conclusions

We have presented preliminary results from an experiment conducted to investigate the validity of the elastic plate model for ice floes in the case of wave heights significant enough to cause overwash. We have show that a simplified two-dimensional linear models gives reasonable agreement with measured results. The next stage in this project is to compare with the three-dimensional theory developed in [5].

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References


