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The following are brief descriptions of *Mathematicians in Schools* sessions with Year 4 and 5 students at Belair Primary School in 2010-2013. Each session is about 40 minutes and is focussed on an important concept that I try to make accessible to the students. I try to direct them to individually or as a group discover something themselves. Each session includes an activity for the students. Each topic has been successfully classroom-tested at least twice; half of them have been tested four times.

Prime numbers. Discover them through a factorisation game. Confirmed that the students know the multiplication table very well. Did the sieve of Eratosthenes up to 99. Got four students to come up and cross out multiples of 2, 3, 5, 7. All understood why we don't need to consider 4, 6, 8, 9; it's a little harder to explain why we can stop there. Talked about patterns in the primes being mysterious, twin primes, infinitude of natural numbers. Stated the infinitude of primes (mentioned Euclid) and the twin primes conjecture.

Angles and how to add them. General discussion of angles and how to add them. Discover that the angles of a triangle add up to a straight angle by cutting a couple of triangles out of paper and adding their angles by placing them side by side. Whereas the weaker students find some of my other topics difficult, everyone can easily do this one. What about a quadrilateral, pentagon, etc.? Finished with a quick game: make as many numbers as possible out of 1, 2, 3, 4 by inserting arithmetic operations.

Seven bridges of Königsberg. Explained the problem, drew the river, islands, and bridges, and then a schematic picture as a network. Introduced the term "circuit". Students grasped the problem and started looking for a circuit, without success of course, since one doesn't exist. I removed one bridge and then added a new bridge, giving the students two more networks to investigate. The third network was easily found to have a circuit. I introduced the notion of the degree of a node and we worked out the degrees of the nodes of the three networks. I asked: how is the network for which we found a circuit different from the others. Fairly quickly came the observation that for the network with a circuit, all the nodes have even degree. I then told them of Euler's discovery that a network has a circuit if and only if each node has even degree.

The binary system. Reviewed the decimal system and place-value notation. Then introduced the binary system. Explained replacing the "building blocks" 1, 10, 100, ... by 1, 2, 4, 8, ... Pointed out that in the binary system we only need one block of each size. Gave them a list of numbers to convert from decimal to binary notation, and asked them to pick some numbers themselves. Some students were quick to understand binary notation, others not so, but almost all understood it eventually. We collected their numbers on the smartboard. One student converted 10,000 into binary! Finally, showed them how to count to 1023 on your fingers.

The four colour problem: the mathematics of colouring in. Explained the rules of colouring maps: adjacent countries must have different colours; use as few colours as possible. Coloured two maps on the board, then gave the students three more to do themselves. This is a very accessible activity. Students came up to the board to show how they'd coloured them. All five maps could be done with 3 or 4 colours. Asked: Is there a map that requires 5 colours? Students took some time to look for one. Some came up with complicated maps that seemed to require 5 or even 7 colours. I suggested they could be coloured with fewer colours. Finally, told them about the four colour problem, but did not give away the solution. The teacher suggested that the students could keep working on it. Some did, as I found out later.

Modular arithmetic. There are ways to do multiplication that only require a few numbers. Showed them the multiplication table modulo 5. Asked them to try to figure out the rule as I slowly filled in the table. Started them on the table for 6. They each completed their own table. We talked about the patterns in the tables and the differences between them. The main differences are explained by 5 being prime and 6 being composite, as at least a couple of students realised. Finally, mention applications of modular arithmetic in cryptography.

Euler's formula for connected planar networks. Introduced terminology. Each student drew one or two planar networks and counted vertices, edges, and faces (they need to be encouraged not to make their networks too large!). We made a table of v, e, f for the students' networks. Asked if they could spot any patterns. Many suggestions. After some discussion, two students spotted the relationship $v-e+f=1$. Rather than explain a proof of Euler's formula, which didn't work so well the first time I did this topic (it required too much concentration at the end of the session), I drew two simple networks on a sphere and a torus. We worked out $v-e+f$ and noted that the three surfaces have different Euler characteristics. A precise way to tell the sphere and the torus apart!

The number pi. Recalled areas of rectangles, then inscribed a circle into a square of side length 20. Asked about the area of the circle. Noted that it must be between 200 and 400. Students thought it would be about 300. We started the activity of counting the unit squares inside a circle of radius 10. The teacher had prepared such a circle for each student on a piece of 5mm graph paper in advance. We discussed the problem of estimating partial squares on the boundary. The kids always plunge into this activity very enthusiastically. There are always several students who get between 310 and 320. The closest one (sometimes 314) receives applause! Finally, handed out a sheet with the first 6,571 digits of pi. The students found this fascinating. This is probably my most fun topic.